What Discounted Cash Flow Rate of Return Never Did Require

E.L. Dougherty, SPE, U. of Southern California

Summary. A misunderstanding exists on the reinvestment of cash flows from a project to meet a calculated rate of return for an investment. Several textbooks state that cash flows from a project must be reinvested at the calculated rate of return in order for that cash flow rate of return for an investment to be realized. We demonstrate mathematically and logically that such reinvestment is not required.

Introduction

For an investment to yield the calculated discounted cash flow rate of return for an investment, i_r, must the cash flows be reinvested from receipt to the project's end at i_r? The correct answer is "no." But a recent article[1] cites 10 frequently used financial textbooks that teach the incorrect interpretation. Confusion occurs because if i_r is to be earned on the initial investment, the correct answer is "yes." This requirement is sensible only when a single cash flow occurs at the project's end, such as with zero coupon bonds, so that reinvestment is impossible. For any project with cash flow before its end, the sensible requirement is for i_r to be earned on the unamortized investment, in which case the disposition of the cash flows is not a consideration.

This confusion pervades SPE literature. At a recent symposium, while I was proving the correct answer mathematically,[2] two other speakers were stating the contrary.[3,4] Boyle and Schenck[4] state, "IRR (i_r) ...assumes that all cash thrown off by a project will be reinvested at the high IRR of that project. Because such reinvestment is unlikely,...projects with a high IRR...will return significantly less than is forecast...." My mathematical demonstration is repeated here, and the correct interpretation illustrated with an example.

Continuous Discounting Procedures

The use of continuous discounting simplifies the mathematical demonstration. Discounting formulas measure economic equivalence between cash flows at different times while taking into account interest earned during the intervening time. With interest compounded continuously at rate i, the rate of increase in wealth, (future worth) is given by

$$dP_{Ft}/dt = iP_{Ft}. \quad (1)$$

The solution to Eq. 1 giving P_{Ft} at any time, t, is

$$P_{Ft} = P_{F0}\exp(it). \quad (2)$$

Multiplying Eq. 2 by $\exp(-it)$ gives the present worth of P_{Ft} at $t=0$:

$$P_{PV} = P_{F0} = P_{Ft}\exp(-it). \quad (3)$$

Thus, if P_{Ft} is a lump sum cash flow at t, the discount factor that gives its present worth is $\exp(-it)$. With a uniform and continuous payment of P, the rate of increase of future worth is given by

$$dP_{Ft}/dt = iP_{Ft} + P. \quad (4)$$

From Eq. 4, when P is received from $t=0$ to $t=t$, P_{Ft} is given by

$$P_{Ft} = (P/i)\,[\exp(it) - 1]. \quad (5)$$

From Eq. 3, it follows that the present worth of this payment is

$$P_{PV} = P_{Ft}\exp(-it) = (P/i)[1 - \exp(-it)], \quad (6)$$

so that the discount factor that gives present worth of a continuous payment at its start is $[1-\exp(-it)]/i$.

Rate of Return and Present Worth Equation

With a single investment (C_i, at $t=0$), the all-important net present value (P_{NPV}) is given by

$$P_{NPV} = -C_i + P_{PV}. \quad (7)$$

The discount rate for which $P_{NPV}=0$ is i_r. The numerical value of i_r depends upon the discounting method, but this variation need not concern us if the same discounting procedure is used with all projects.

Meaning of Rate of Return

We use the cumulative cash balance, C_{Bt}, to demonstrate that i_r is the earnings rate on the unrecovered portion of the investment regardless of what is done with the project's cash flows. C_{Bt}, the unamortized investment at any time, equals the future worth at t of the project's discounted cumulative cash flow at t. The following example with $i_r=0$ illustrates C_{Bt}. Suppose an investment ($C_i=\$1{,}000$) yields $P=\$500$/yr for 2 years, so that at $t=2$, C_i is returned with no interest.

Clearly, $C_{B0}=-C_i=-\$1{,}000$, and $C_{B2}=0$. For this example, Eq. 4 becomes

$$dC_{Bt}/dt=P, \quad (8)$$

and $C_{B0}=-C_i=-\$1{,}000$, and $P=\$500/\text{yr}$. The solution to Eq. 8 is

$$C_{Bt}=C_{B0}+Pt=-1{,}000+500t, \quad (9)$$

and $C_{B2}=0$. Eq. 9 illustrates that C_{Bt} is the unamortized investment at t. A graph of C_{Bt} vs. t is a straight line from $(0,-1000)$ to $(2,0)$.

In the general case with $i_r>0$, Eq. 8 is replaced by

$$dC_{Bt}/dt=iC_{Bt}+P, \quad (10)$$

and $C_{B0}=-C_i$. Initially, $iC_{Bt}<0$, because $C_{Bt}<0$. If $P>|iC_{Bt}|$, the right side of Eq. 10 is positive, so that P is sufficient to increase C_{Bt}—i.e., reduce the unamortized investment—while paying interest at rate i on the balance. The solution to Eq. 10 is

$$C_{Bt}=-C_i \ \exp(it)+P[\exp(it)-1]/i. \quad (11)$$

Eqs. 2 and 5 show that the terms on the right side of Eq. 11 are the future worth at t of $-C_i$ and of total cash received (Pt), respectively. From Eq. 3, the present worth of C_{Bt} with $t=n$ is

$$C_{Bn} \ \exp(-in)=-C_i+P[1-\exp(-in)]/i. \quad (12)$$

The right side of Eq. 12 equals P_{NPV} at discount rate i of an initial investment (C_i) that yields continuous cash flow of P from $t=0$ to $t=n$. Thus, setting the right side of Eq. 12 equal to zero gives $P_{\text{NPV}}=0$, and the value of i satisfying this equation is i_r. Hence, we can write

$$C_{Bn} \exp(-i_r n)=P_{\text{NPV}}$$

$$=-C_i+P[1-\exp(-i_r n)]/i_r=0. \quad (13)$$

This completes our demonstration of i_r's meaning. Eqs. 12 and 13 show that i_r is the interest rate for which C_i is fully amortized—i.e., $C_{Bn}=0$—and Eq. 10 shows that interest is earned only on C_{Bt}, the unamortized investment when $t<n$.

Why the Interpretation is Incorrect

Multiply Eq. 13 by $\exp(i_r n)$ and rearrange the result to obtain

$$C_i \ \exp(i_r n)=P[\exp(i_r n)-1]/i_r. \quad (14)$$

The left side of Eq. 14 is the future worth of C_i if it earns interest at i_r compounded continuously from $t=0$ to $t=n$, and the right side is, by Eq. 5, the future worth of the reinvested cash flow (Pn), if between $t=0$ and $t=n$ the current balance is earning interest compounded continuously at i_r. Thus Eq. 14 states that if i_r were to be realized on the initial investment, all the project's cash flows must be reinvested at this same earnings rate.

The interpreters may mistakenly assume that Eq. 14's statement of equivalence relates to investment decision-making; it does not. The essence of investment performance is contained in Eq. 14's equivalent form:

$$C_i=P[1-\exp(-i_r n)]/i_r. \quad (15)$$

As Eqs. 10 and 12 show, Eq. 15 requires i_r to be earned on the unamortized investment. Eq. 15's requirement is consistent with the responsibilities accepted by a corporate manager to whom investment funds are allocated. These funds are invested, and performance is judged upon how closely actual cash flows approximate projections. The manager does not control what is done with the project's cash flow. These are gathered into the corporate treasury and used as the corporate executive sees fit. The manager is accountable only for the unrecovered investment funds he is actually using, which are targeted to earn i_r. To hold a manager accountable for the earnings rate of cash flows returned to the treasury is ridiculous. Any executive staff that proposed such a performance evaluation would not be taken seriously. No shrewd manager will agree to being evaluated on the performance of something over which he has no control.

The following example personalizes the difference between the two interpretations. One of our most important decisions is whether to buy a home. Suppose we take out a 30-year mortgage for \$50,000 at 12%/yr. Assume interest on the unamortized balance is compounded continuously, and that we are not charged points for the loan. From Eq. 6, the annual payment is $P=(0.12)(50{,}000)/[1-\exp(0.12\times30)]=\$6{,}168.55/\text{yr}$, for a monthly payment of \$514.05. This value assumes we are not charged interest intramonthly while the continuous payment accumulates to \$514.05; if interest were charged until the moment of payment, then from Eq. 5, our actual monthly payment would be \$516.62. The total amount we contract to pay the lender is $(30)(12)(514.05)=\$185{,}085$.

Instead, suppose that we guarantee the lender 12% interest on his \$50,000 for 30 years. The amount guaranteed is $(50{,}000)\exp(0.12\times30)=\$1{,}829{,}912$. From Eq. 14, note that $(6{,}168.55)[\exp(0.12\times30)-1]/0.12]=\$1{,}829{,}912$, so that if the lender reinvests the monies we pay him at 12%, the future worth at $t=30$ of the cumulative payments equals \$1,829,912, and our contracted liability just fulfills the guarantee. If, instead of reinvesting the payments, the lender spends them frivolously, then to fulfill our guarantee we must come up with $\$1{,}829{,}912-185{,}085=\$1{,}644{,}827$ at $t=30$. The difference between these two forms of liability is the housing industry.

This paper should prove that i_r does not suffer from the shortcoming so often attributed to it. Rather, the fault is a fixation by the critics on a mechanistic transformation of discounting formulas without examining the formula's meaning carefully.

Nomenclature

C_B = cash balance, dollars
C_{Bn} = cumulative cash balance at $t=n$, dollars
C_{Bt} = cumulative cash balance in a project at any t, dollars

C_{B0} = cumulative cash balance at $t=0$, dollars
C_{B2} = cumulative cash balance at $t=2$, dollars
C_i = initial investment at $t=0$, dollars
i = discount rate or interest rate, %/yr
i_r = discounted cash flow rate of return for an investment, %/yr
n = life of an investment project, years
P = rate of cash flow from an investment, dollars/yr
P_{Ft} = future worth of some amount of money, dollars
P_{F0} = an amount of money at $t=0$, dollars
P_{NPV} = discounted net present value of an investment project, dollars
P_{PV} = present worth at $t=0$ of some amount of money, dollars
t = time, years

References

1. Beidleman, C.R.: "Discounted Cash Flow Reinvestment Rate Assumptions," *Eng. Economist* (1984) **29**, No. 2, 123–39.
2. Dougherty, E.L.: "Guidelines for Proper Application of Four Commonly Used Investment Criteria," paper SPE 13770 presented at the 1985 SPE Hydrocarbon Economics and Evaluation Symposium, Dallas, March 14–15.
3. Terry, R.E. and Ehman, M.P.: "Internal Rate of Return: Friend or Foe?" paper SPE 13771 presented at the 1985 SPE Hydrocarbon Economics and Evaluation Symposium, Dallas, March 14–15.
4. Boyle, H.G. Jr. and Schenck, G.K.: "Investment Analysis: U.S. Oil and Gas Producers Score High in University Survey," *J. Pet. Tech.* (April 1985) 680–90.

JPT

Original manuscript (SPE 14694) received in the Society of Petroleum Engineers office July 26, 1985. Paper accepted for publication Nov. 18, 1985. Revised manuscript received Oct. 28, 1985.

SPE 15327

Discussion of What Discounted Cash Flow Rate of Return Never Did Require

Cheryl R. Collarini, SPE, Collarini, Toal & Assocs. Inc.

Dougherty's paper, "What Discounted Cash Flow Rate of Return Never Did Require" (Jan. 1986 *JPT*, Pages 85–87) states that the cash flows from a project need not be reinvested at the calculated internal rate of return (IRR) for that project to yield that rate of return (ROR). While the mathematical proof presented is correct, the application to petroleum investment decisions is not.

The author states that the "sensible requirement is for i_r to be earned on the unamortized investment, in which case the disposition of the cash flows is not a consideration." The mathematical demonstration presented does indeed prove that i_r is actually earned on the unamortized investment; however, it does not serve to eliminate the "confusion [that] pervades SPE literature."

The discounted cash flow (or internal) ROR is simply a measure of project efficiency that is more useful in the banking industry than in the petroleum business. Most companies and individuals are targeted to maximize dollars, not efficiency. IRR is used as the decision-making criterion because of its familiarity to the average manager rather than its validity as a profit-maximizing indicator. It is not always true that decision-makers have a thorough understanding of financial analysis as it relates to profitability indicators; the majority, however, will understand the concept of ROR from dealing with banks in their personal lives. The adjusted (or reinvestment)[1] ROR can be used as a decision criterion that is both consistent with maximizing value and familiar to most decision-makers.

With an unlimited budget (more money than investment opportunities), the investor need only decide whether to accept or to reject each project. All projects that yield an IRR greater than the cost of investment capital are acceptable. Because the IRR is defined as the discount rate where present value is equal to zero, decisions made on an ROR cutoff will always be consistent with adding present value. Accepting every available opportunity with a positive present worth will maximize the firm's value. In this situation, the author is correct in stating that the reinvestment rate of the interim cash flows is irrelevant.

With a limited budget (more opportunities than money to invest), each opportunity must be compared to other available opportunities to select the combination of projects that will maximize the value to the firm. The author's conclusions do not apply in this situation. The following simple examples demonstrate how the use of the IRR can lead to investment decisions that are inconsistent with maximizing dollars. Table D-1 illustrates three investment opportunities that have a calculated IRR of 25% with projected cash flows over 5 years.

If the cost of capital is 12% and there is an unlimited budget (at least $3,000), the correct decision would be to invest in all three opportunities; each has an ROR greater than 12% and, consequently, a positive present worth at 12%. If you have a limited budget, however, and your average opportunity rate is expected to be about 15% over the life of these projects, the decision is not so simple. In ranking these opportunities, the primary criterion is maximizing the dollar amount at the end of 5 years. Project A's $400 annual cash flow can be invested at 15% and compounded continuously to yield $2,761 at the end of 5 years. Project C will result in a cash total of $3,301,

TABLE D-1—THREE INVESTMENT OPPORTUNITIES

Year	A (dollars)	B (dollars)	C (dollars)
0	1,000	1,000	$1,000
1	400	—	—
2	400	—	—
3	400	—	—
4	400	—	1,550
5	400	3,490	1,500

TABLE D-2—SIX PROJECTS RANKED FOR BUDGET PRIORITY

Year	A (dollars)	B (dollars)	C (dollars)	D (dollars)	E (dollars)	F (dollars)
0	10,000	10,000	10,000	10,000	10,000	10,000
1	—	—	1,000	—	—	—
2	—	6,000	3,000	—	—	—
3	—	8,000	5,000	7,000	—	—
4	—	5,000	4,000	7,000	10,000	—
5	—	—	4,000	6,000	7,000	15,000
6	31,645	—	3,000	6,000	7,000	14,000
IRR, %	19.2	22.3	19.2	22.3	18.2	19.5
Present worth, dollars	7,367	4,190	3,819	6,810	4,790	6,781
Adjusted ROR, %	19.2	15.8	15.4	18.7	16.5	18.6

*10% discount rate assumed.

and Project B in $3,490. Although all three projects have equal IRR's, they result in various amounts of cash dollars. The reason is that Project B's entire cash flow yields a compounded 25% ROR throughout its life, as does the majority of Project C's cash flow. The adjusted ROR's, using the end-of-life cash flows, are 20.3%, 25%, and 23.9% for Projects A, B, and C, respectively. These reinvestment ROR's are consistent with the actual dollars expected to be received.

Table D-2 illustrates six projects in the corporate inventory that are ranked for budget priority. Ranking these projects by use of the three criteria yields the priority listings in Table D-3.

The order in which these projects would be accepted is the same under the adjusted ROR and present-worth criteria but is different from the IRR ranking. Under a budget constrained to $40,000, the present worth at 10% would be $21,600 with the IRR as the decision criterion. Ranking on ROR, the present worth would be $25,748. It is clear that not only is the reinvestment rate a factor, but it is a critical factor in making the right investment decisions.

While the author states that "to hold a manager accountable for the earnings rate of cash flows returned to the treasury is ridiculous," it is equally ridiculous to maintain a management organization that has no knowledge of the financial performance of the corporation. Managers should be selected on the basis of their ability to make decisions that enhance the corporation's value over the long term. The most important part of any decision is the assimilation of all relevant information regardless of personal accountability. Ignorance of the average opportunity rate is no better excuse for making suboptimum decisions than is ignorance of the lease burden when computing cash flows. While it is agreed that in a large corporation, managers will rarely be caught making a less-than-optimum decision, the best managers will consider all factors before committing funds. These factors should include estimating the expected average reinvestment rate. As for myself, I don't particularly care about the mathematical proofs as long as I have the most dollars when the game is over.

TABLE D-3—PRIORITY LIST

IRR	Present Worth	Adjusted ROR
B	A	A
D	D	D
F	F	F
C	E	E
A	B	B
E	C	C

Reference

1. Starks, L.: "Economic Performance of Oil Projects Evaluated by Modified IRR," *Oil & Gas J.* (April 22, 1985) 71-74.

(SPE 15327)

JPT

Discussion of What Discounted Cash Flow Rate of Return Never Did Require

SPE 15324

R.S. Wansbrough, SPE, Petro Logic Co.

Dougherty's disagreement with critics[1] of the discounted cash flow (DCF) rate-of-return (ROR) calculation is based on a misapprehension of their criticisms. His mathematical demonstration provides proof that the method is consistent with its own definition, a truism, and it could not possibly be otherwise. The logical argument is a nonsequitur because nothing in the definition confines its applicability to managers, executives, or management theory. The criticisms are directed at its usefulness.

A demonstrable flaw in the DCF method is its logical inconsistency with the legitimate idea of present value, a perfectly unambiguous concept. The present value of income to be received should be determinable by any rational entity. In the case of the DCF method, two cash flows of substantially different ROR's result in an interesting paradox. For example, consider cash flows with respective ROR's of 15 and 40%. These will yield substantially different discounted values per dollar year by year into the future. These values are used *as if* they were actual present values. In the 10th year, for example, cash received from the lower-return project is evaluated as having a present value per dollar that is approximately seven times greater than that of the higher return. This requires an investor to entertain contradictory evaluations of the present values of equivalent future dollars simultaneously. This is logical nonsense and an obvious paradox.

Authors Boyle and Schenck are entitled to complain that the English-language expression "rate of return" related to a project misleadingly implies that the project will yield that ROR on the investment amount for the life of the project. However, it is not clear to all investors that this is not the case. Boyle and Schenck's cautionary words are worth heeding.

Reference

1. Boyle, H.G. Jr. and Schenck, G.K.: "Investment Analysis: U.S. Oil and Gas Producers Score High in University Survey," *JPT* (April 1985) 680-90.

(SPE 15324)

JPT

SPE 15331

Discussion of What Discounted Cash Flow Rate of Return Never Did Require

Roberto Aguilera, SPE, Servipetrol Ltd.

I congratulate Dougherty on his recent paper. I strongly agree with his comments. I tried to get the same point across in SPE literature some 10 years ago when I stated that most petroleum engineering articles dealing with economic analysis claim that reinvestment of incomes is implied by the rate of return (ROR) calculation. This is a misleading concept. An investor is interested in the ROR for a specific project investment, regardless of how he spends, wastes, or reinvests his income. The ROR gives the return that an investor receives on his unamortized investment.[1] The same principles have been stated previously by Stermole.[2]

If the assumption is made that incomes are received at the end of each year, the concept discussed here can be written as follows.

For the first year,

$$C_1 = -C-(C\times i) \quad \text{(D-1a)}$$

and

$$C_{u1} = C_1 + P_1. \quad \text{(D-1b)}$$

For the second year,

$$C_2 = C_{u1}(1+i) \quad \text{(D-2a)}$$

and

$$C_{u2} = C_2 + P_2. \quad \text{(D-2b)}$$

For the third year,

$$C_3 = C_{u2}(1+i) \quad \text{(D-3a)}$$

and

$$C_{u3} = C_3 + P_3. \quad \text{(D-3b)}$$

. . .

For the nth year,

$$C_n = C_{un-1}(1+i) \quad \text{(D-4a)}$$

and

$$C_{un} = C_n + P_n. \quad \text{(D-4b)}$$

In Eqs. 1 through 4, C_{u1}, C_{u2}, $C_{u3} \ldots C_{un}$ represent the unamortized investment on which the ROR is applied each year, C is the invested capital, and C_1, C_2, $C_3 \ldots C_n$ are the amounts of principal plus interest, i. When the value of C_{un} equals zero, the assumed i equals the ROR.

Table D-1 shows the economics of a secondary recovery project reported by Glanville.[3] The statement that reinvestment of incomes is implied by the ROR calculation is shown to be invalid with the use of this example. By remembering that the ROR gives the return that an investor receives on the unamortized investment, we can enter data from Table D-1 into Eq. 1 as follows.

For the first year,

$$C_1 = -400{,}000-(400{,}000\times 0.1054) = -442{,}160 \quad \text{(D-5a)}$$

and

$$C_{u1} = -442{,}160-100{,}000 = -542{,}160. \quad \text{(D-5b)}$$

For the second year,

$$C_2 = -542{,}160\times 1.1054 = -599{,}304 \quad \text{(D-6a)}$$

and

$$C_{u2} = -599{,}304+0 = -599{,}304. \quad \text{(D-6b)}$$

For the third year,

$$C_3 = -599{,}304\times 1.1054 = -662{,}470 \quad \text{(D-7a)}$$

and

$$C_{u3} = -662{,}470+50{,}000 = -612{,}470. \quad \text{(D-7b)}$$

. . .

For the 16th year,

$$C_{16} = -22{,}816\times 1.1054 = -25{,}221 \quad \text{(D-8a)}$$

and

$$C_{u16} = -25{,}221+25{,}000 = -221 \simeq 0. \quad \text{(D-8b)}$$

These results are plotted in the cumulative-cash-position diagram presented in Fig. D-1. Because we expect to receive 10.54%/yr on unamortized investment, the cumulative-cash position is reduced by 10.54% of the unamortized investment at the beginning of each year. The

TABLE D-1—ECONOMIC EVALUATION, ROR ANALYSIS

Basic Data
Oil price = $10.000/m^3
Rate of discount = 10.00%/yr
First year = 1
Number of years to be analyzed = 16
Solution

Year	Oil (m^3/yr)	Gross Revenues (dollars/yr)	Operating Expenses (dollars/yr)	Operating Income (dollars/yr)	Investment (dollars)	Cash Flow (dollars/yr)
1	0	0	100,000	−100,000	400,000	−500,000
2	10,000	100,000	100,000	0	0	0
3	15,000	150,000	100,000	50,000	0	50,000
4	17,500	175,000	100,000	75,000	0	75,000
5	17,500	175,000	100,000	75,000	0	75,000
6	20,000	200,000	100,000	100,000	0	100,000
7	20,000	200,000	100,000	100,000	0	100,000
8	20,000	200,000	100,000	100,000	0	100,000
9	20,000	200,000	100,000	100,000	0	100,000
10	20,000	200,000	100,000	100,000	0	100,000
11	20,000	200,000	100,000	100,000	0	100,000
12	20,000	200,000	100,000	100,000	0	100,000
13	20,000	200,000	100,000	100,000	0	100,000
14	20,000	200,000	100,000	100,000	0	100,000
15	15,000	150,000	100,000	50,000	0	50,000
16	12,500	125,000	100,000	25,000	0	25,000
Totals	267,500	2,675,000	1,600,000	1,075,000	400,000	675,000

Discounted Profit to Investment Ratio = 0.05
Payout Time = 96.00 months
Payout = 268.75%
Present Worth at 10.00%/yr = $19,452
ROR = 10.54%/yr

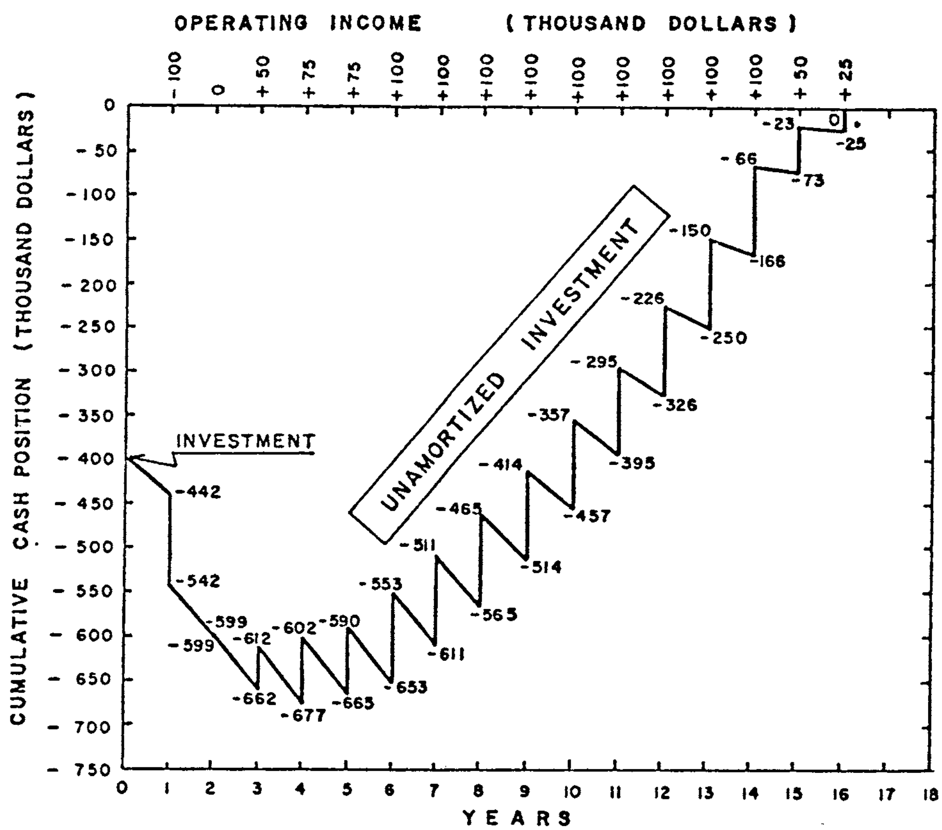

Fig. D-1—Cumulative cash position diagram. Example 1.

annual income is added to the amount of principal obtained each year to generate the new unamortized investment. Note that the ROR can be calculated by trial and error from Eqs. 1 through 4.

Fig. D-1 permits the instantaneous evaluation of unexpected issues. If, for example, we had to terminate this project at the end of the 10th year, we would have to receive $357,436 in addition to the incomes received during the first 9 years to realize a 10.54%/yr ROR.

Notice that the cumulative-cash position is zero at the end of the project if the ROR equals 10.54%/yr. Consequently, we emphasize that this project ROR is 10.54%/yr, no matter how the incomes are reinvested.

I hope that Dougherty's contribution will not be overlooked.

Nomenclature

C = single investment at Time zero, dollars
C_n = amount of principal plus interest at Time n, dollars
C_u = unamortized investment at Time n, dollars
i = period compound interest, fraction
n = number of interest-compounding periods
P_n = cash flow from an investment C at Time n, dollars

References

1. Aguilera, R.: "Economic Analyses of Acceleration Projects," paper SPE 6086 presented at the 1976 SPE Technical Conference and Exhibition, New Orleans, Oct. 3-6.
2. Stermole, F.J.: *Economic Evaluation and Investment Decision Methods*, Investment Evaluations Corp., Golden, CO (1974).
3. Glanville, J.W.: "Rate of Return Calculations As a Measure of Investment Opportunities," *JPT* (June 1957) 12-15.

SI Metric Conversion Factor

bbl × 1.589 873 E−01 = m^3

(SPE 15331)

JPT

Discussion of What Discounted Cash Flow Rate of Return Never Did Require

SPE 15332

M. Wolff, SPE, U. of Texas
W.C.J. Van Rensburg, U. of Texas

In his paper "What Discounted Cash Flow Rate of Return Never Did Require" (Jan. 1986 *JPT*, Pages 85-87), E.L. Dougherty presents a valid mathematical description of the internal rate of return (IRR), but unfortunately it contains some rather overzealous statements in regard to its application. His equations clearly describe the meaning of the IRR; however, his statements that imply that investment decision-making should not consider the reinvestment rate overstate the case. For example, consider the statements "For any project with cash flow before its end, the sensible requirement is for IRR to be earned on the unamortized investment, in which case the disposition of the cash flows is not a consideration" and "the interpreters may mistakenly assume that Eq. 14's statement of equivalence [that to realize an earnings rate equal to the IRR, all cash flows must be reinvested at the IRR] relates to investment decision-making; it does not."

TABLE D-1—COMPARISON OF INVESTMENTS

	Investment A	Investment B
Investment, dollars	10,000	10,000
Year 1 cash flow, dollars	2,000	12,000
Year 2 cash flow, dollars	12,000	1,000
IRR, %	20.0	27.8
Net future worth, dollars	14,100	13,600
GROR, %	18.7	16.6

A simple example should point out some problems with investment without regard to the reinvestment rate. Consider a person with $10,000 to invest for 2 years who must choose between two possible investments, A and B. Assuming that intermediate cash flows are invested in a bank at 5%, the investor must decide which investment is the better one (see Table D-1).

The IRR ranking would show that Investment B is the preferred investment, but Investment A clearly puts more money in pocket at the end of 2 years. The growth rate of return (GROR), which is computed by use of the reinvestment rate, leads to the correct investment choice.

The IRR/reinvestment debate is essentially a semantic one. We agree with Dougherty that the IRR implies reinvestment at the IRR only if we are concerned about actually making a return on investment at that rate. However, because the "essence of investment performance" is maximizing future worth (not just the IRR), it seems clear that investment ranking without regard to reinvestment rate is incomplete.

(SPE 15332)

JPT

Author's Reply to Discussions of What Discounted Cash Flow Rate of Return Never Did Require

SPE 15681

E.L. Dougherty, SPE, U. of Southern California

I thank the authors for their discussions of my paper (Jan. 1986 *JPT*, Pages 85–87) and for providing me the opportunity to offer further clarification of how to interpret discounted cash flow rate of return, i_r. My responses follow.

Aguilera demonstrates the correctness of the interpretation presented in my paper with an example that assumes annual compounding and end-of-year payments. For these assumptions, the cumulative cash balance,* C_{Bj}, is defined only at year-end and is given by

$$C_{Bj}=-C_j(1+i_r)^j+\sum_{k=1} P_{Fk}(1+i_r)^{j-k},\ 1\leq j\leq n,$$

$$=-C_i(1+i_r)^j+P_{F1}(1+i_r)^{j-1}$$

$$+P_{F2}(1+i_r)^{j-2}+\ldots+P_{Fj}. \qquad \text{(R-1)}$$

If P_{Fk} is a constant independent of time, Eq. R-1 becomes

$$C_{Bj}=-C_i(1+i_r)^j+P_F\sum_{k=1}^{j}(1+i_r)^{j-k}$$

$$=-C_i(1+i_r)^j+P_f[(1+i_r)^j-1]/i_r. \qquad \text{(R-2)}$$

Eq. R-2 is the analog of my paper's Eq. 11. If $C_{Bn}=0$, the analog of my original Eqs. 13 and 14 is obtained. In Aguilera's example, $n=16$ and P_F changes with time, so Eq. R-1 must be used. The result is

$$C_{Bn}=0=-400(1+i_r)^{16}-100(1+i_r)^{15}+50(1+i_r)^{13}$$

$$+\{75[(1+i_r)^2-1]/i_r\}(1+i_r)^{11}$$

$$+\{100[(1+i_r)^9-1]/i_r\}(1+i_r)^2+50(1+i_r)+25,$$

$$\qquad \text{(R-3)}$$

where the solution is $i_r=10.54\%$/yr, as originally given.

*In the original paper, the subscript *t* was used to denote a continuous variable; here *j* is used to denote time as a discrete variable.

Aguilera correctly credits Stermole[1] with properly explaining the i_r reinvestment question. On three occasions at major SPE conferences, I have suggested during discussion of technical papers that authors and other commentators read Sec. 3.11 in Stermole. When I presented the arguments advanced in a paper in Dallas,[3] however, the author who followed me argued exactly the opposite. This somewhat startling occasion convinced me that the issue deserved a wider consideration than afforded by my presentation.

Because Collarini's and Wolff and Van Rensburg's discussions overlook that my paper dealt only with the interpretation of i_r, these comments are directed mainly at things I did not say. Specifically, I in no way dealt with growth rate of return, i_g, how it differs from i_r, and the benefits from using i_g, all of which are central issues to these comments. Thus, Collarini's remark prefacing the discussion of i_g that "The author's [my] conclusions do not apply in this situation" is not a critique, but merely a statement of fact. Likewise, the concluding sentence in Wolff and Van Rensburg's discussion, "However, because the 'essence of investment performance' is maximizing future worth (not just the IRR), it seems clear that investment ranking without regard to reinvestment rate is incomplete," expresses an acceptable opinion about decision criteria, but it does not relate to my paper. When these commentators focus on the matter I discussed, it is gratifying that they agree with its conclusions. As Collarini says, "In this situation, the author is correct...that the reinvestment rate of the interim cash flows is irrelevant." Wolff and Van Rensburg "agree...the IRR implies reinvestment at the IRR only if we are talking about...making a return on investment at that rate."

Even while recognizing the broad scope of these two comments, let us see what they can contribute to the reader's understanding of economic decision criteria. Recall that with a single investment, C_i, at $t=0$ with continuous discounting, i_g is given by

$$i_g=(1/n)\ \ln(P_{Fn}/C_i), \qquad \text{(R-4)}$$

and with annual compounding by

$$i_g=(P_{Fn}/C_i)^{(1/n)}-1, \qquad \text{(R-5)}$$

where P_{Fn} is the future worth of all of the project's cash flows compounded to $t=n$ with the investor's discount rate, i_d. Collarini uses Eq. R-4; Wolff and Van Rensburg use Eq. R-5. Both assume end-of-year payments. Note that Collarini calls i_g "adjusted rate of return," an unfortunate deviation from today's common usage. A major theme of each discussion is that i_g can be used as a ranking criterion for capital allocation—i.e., it provides a simple procedure for selecting the set of projects whose combined NPV is the maximum of all possible combinations.

Contrary to the intuition of many, ranking by i_r does not yield the proper selection. Stermole[1] and Capen *et al.*[2] carefully develop these principles. (Because I felt that these two references said about all there was to say about using i_g for capital allocation, I elected not to discuss the matter in the paper presented in Dallas[3] that triggered the current discourse.) Given the thrust of these two comments, perhaps these ideas are not fully understood by the broad base of SPE members, so reaffirming them here is worthwhile.

Simply stated, to allocate a fixed total of funds between alternative investment portfolios, rank the projects by i_g—i.e., the largest i_g first, the next largest second, etc. Picking projects in descending order from this list until all funds are expended yields the NPV-maximizing group. The portfolio considered in Wolff and Van Rensburg's discussion contains only two projects, but because the implied budget limit will allow only one to be undertaken, the allocation rule still applies.

Let us now consider the more subjective thrusts of these comments. Wolff and Van Rensburg suggest that my paper "unfortunately. . .contains some rather overzealous statements." Viewed in the narrow context they were intended to serve (to put to rest once and for all the often-cited misconception about i_r), I believe the statements are most appropriate. In the broader context of the comment, this suggestion may have some merit.

The faults alleged in Collarini's discussion are more intriguing, sprinkled as they seem to be with the sauce of controversy. As a start, I confess I am unable to pin down the exact nature of the complaints. Particularly, the forceful condemnation, "While the. . .proof. . .is correct, the application. . .is not," seems to object to things the author imagined were implied rather than things actually written. As noted above, the comment agrees that in the assessment of i_r, the reinvestment rate "is irrelevant." But if this is so, then why declare that "it [mathematical demonstration] does not serve to eliminate the 'confusion'" about i_r? And because my paper dealt only with i_r, why is it that "the application to petroleum investment decisions is not [correct]"?

I fully believe that "to hold a manager accountable for the earnings rate of cash flows returned to the treasury is ridiculous." Consider a hypothetical situation. Suppose a major stockholder decides peremptorily to fire the two top officers for inept performance. Should the manager of the major producing division, who diligently plied his trade to generate cash flow whose disposition was the two officers' undoing, also be dismissed? Find me one rational professional who will answer "yes"! I certainly agree that "it is equally ridiculous to maintain a management organization that has no knowledge of the financial performance of the corporation." It seems somewhat senseless to hypothesize such an organization and to presume that it makes investment decisions without performing economic evaluations with an officially designated discount rate thought to approximate the actual average opportunity rate. All corporations that I have contact with (including those employing graduates I have taught economic evaluation principles and procedures) use discounted cash flow. Hence, whether they choose to recognize it or not, they are deploying the principles underlying growth rate of return. The basis for this last statement is succinctly stated by the mathematical equation relating i_g, the discount rate i_d, project life n, and the project's present value ratio[2,3]

$$i_g = i_d + (1/n)\ \ln[(P_{NPV}/C_{PW}) + 1], \quad \text{(R-6)}$$

where P_{NPV} is the discounted net cash flow and C_{PW} is the discounted investment. Perhaps if this equation's existence were more widely known, less confusion would exist about how i_g fits into the economic evaluation spectrum. Once again, we could take our hats off to the beauty of that one universal language, mathematics, and admire its ability to convert confused babbling into coherent discussion.

Wansbrough presents some confounding arguments. To begin, he declares that my "disagreement with critics. . .is based on a misapprehension of their criticism." Because my Funk and Wagnall's dictionary doesn't define "misapprehension," my alleged basis is unclear. Maybe I don't understand the nature of the fault critics find with i_r? Not true. I understand perfectly what they are saying, and my paper points out why they are wrong. Pushing on through the first paragraph of Wansbrough's discussion still leaves me in the dark, so I conclude that the author was figuratively clearing his throat in preparation for advancing arguments given in his 1960 paper. There, the quantity $[(P_{NPV}/C_{PW})+1]$ is called return on investment, which Eq. R-6 shows is closely related to i_g.

Wansbrough's second paragraph suggests that an investor may be entertained with "logical nonsense and an obvious paradox." This misconception results from indiscriminately mixing two separate concepts. The first concept he covers is present value of an increment of income received at a future time, t. This value hinges on i_d, the average earnings rate at which funds are assumed to be invested. The present value of the increment is the amount of money, which if invested at $t=0$ at i_d would compound to an amount equal to the increment of income at t. Note that in this context, present value in no way involves any statement about investment and can be applied equally well to an income or an expense.

The second concept builds on the first to construct a model of an investment and the resulting cash flow and to define a measure, i_r, of the investment's efficiency. The definition can be thought of as the following hypothetical game. Suppose that each increment of cash flow received consists of a fraction of initial investment plus the compound interest accrued on the investment fraction from $t=0$ until the increment is received. Expressed mathematically for a project with initial investment, C_i, at $t=0$ and uniform cash flow, P, from $t=0$ to $t=n$, the life of the project, we have

$$P\mathrm{d}t = (\mathrm{d}C_i)e^{it}. \quad \text{(R-7)}$$

Rearranging and integrating over the project's life gives

$$\int_0^n \mathrm{d}C_i = C_i = \int_0^n e^{-it} P\mathrm{d}t = (P/i)[1-e^{-in}]. \quad \text{(R-8)}$$

The value of i satisfying Eq. R-8 is i_r, the rate of return on the unamortized investment. The second paragraph of Wansbrough's discussion disregards the hypothesis that led to i_r, uses i_r to calculate present value, and hangs the same conceptual meaning on this present value as on that obtained with the investor's discount rate. As it would be if you were to watch Rambo and the Dirty Dozen simultaneously on the same screen, randomly mixing the signals makes it extremely difficult to tell what is going on.

In summary, putting aside the intricacies introduced by the tax code and by the existence of complex deals for splitting revenues and costs, I think that many engineers find economic evaluation methodology a bit confounding. One difficulty is conceptual. In spite of all the things that are written about it, the meaning of discounted cash flow is elusive. As suggested above, a useful interpretation is obtained by writing Eqs. R-7 and R-8 with the investor's discount rate, i_d, and a hypothetical investment, C_i':

$$P\mathrm{d}t=(\mathrm{d}C_i')\exp(i_d t) \quad \text{(R-9)}$$

and

$$C_i'=\int_0^n \exp(i_d t)P\mathrm{d}t=(P/i_d)[1-\exp(i_d n)]. \quad \text{(R-10)}$$

If $C_i'>C_i$ (the actual investment), then the cash flow returns principal plus interest compounded at i_d for a hypothetical investment greater than the actual investment. Thus, relative to the targeted rate of return, the investor realizes an increase in wealth of $C_i'-C_i$.

A second difficulty is the seemingly large number of economic evaluation yardsticks one encounters. These criteria have different properties and are applicable in a variety of ways. Studying the mathematical properties of these criteria, as well as working through examples, is an effective way to put this spectrum into focus.

Finally, let us hope that aside from being stimulating (at least for me), our examination of economic decision criteria is in some small way helpful to our industry's decision makers. They have a tough job that just got tougher.

References

1. Stermole, F.J.: *Economic Evaluation and Investment Decision Methods,* fifth edition, Investment Evaluations Corp., Golden, CO (1984) 57-60, 115-26.
2. Capen, E.C., Clapp, R.V., and Phelps, W.W.: "Growth Rate—A Rate-of-Return Measure of Investment Efficiency," *JPT* (May 1976) 531-43.
3. Dougherty, E.L.: "Guidelines for Proper Application of Four Commonly Used Investment Criteria," paper SPE 13770 presented at the 1985 SPE Hydrocarbon Economics and Evaluation Symposium, Dallas, March 14-15.

(SPE 15681) **JPT**

SPE 15910

Discussion of What Discounted Cash Flow Rate of Return Never Did Require

J.B. Russell, SPE, Mobil Oil Corp.

Dougherty's paper, "What Discounted Cash Flow Rate of Return Never Did Require" (Jan. 1986 *JPT*, Pages 85–87) has touched an apparently still-sensitive nerve.

The dispute can be waged with mathematical equations and statements of investment goals. Because all the equations are valid and all the goals worthy, the debate will be endless unless we realize that different tools are used to reach different goals.

For those who want to keep it simple, I offer the following observations.

1. No single profit indicator can do everything. Some are dollar values that measure size (profit and present value), and others are ratios that measure efficiency [discounted cash flow (DCF), rate of return (ROR), profit/investment, and present value/investment].
2. DCF ROR is not a ranking tool for budget projects; rather, it is an excellent accept/reject indicator. All projects with a DCF ROR greater than the opportunity cost (i.e., the average reinvestment earning power) are eligible to be accepted; all with less should be rejected.
3. If the budget is limited and your goal is to maximize present value, then the handiest ranking tool for a list of acceptable independent budget projects is the present value/investment ratio—discounted at the reinvestment opportunity rate. This will lead to a budget with the most present value dollars for a given number of investment dollars.
4. Because maximizing present value is the same as maximizing net future worth, it is unnecessary to convert DCF ROR to a growth ROR for ranking purposes. Those who wish may do so and will get the same project selection list as those who rank with the present value/investment ratio.

Debate about whether the DCF ROR implies reinvestment tends to be futile because both the pro and con hinge on the same point: the DCF ROR has a limitation. This limitation is most easily overcome not by modifying ROR to arrive at a new indicator, but by using one of the other indicators already available.

In summary, DCF ROR is a measure of project investment efficiency, not a measure of composite earnings growth. Properly used as a stand-alone accept/reject indicator for individual projects, it does not imply reinvestment.

Because the DCF ROR of a project is not connected with reinvestment, it cannot be used to rank projects for earnings growth. For that purpose, we must use either the present value/investment ratio or a growth ROR. These indicators do use the opportunity reinvestment rate, and of the two, present value/investment is more readily available and less apt to be a source of confusion.

(SPE 15910) **JPT**