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Impact of Investment Criteria, and Reservoir Characteristics, Taxes, Royalty, and Price on Optimal Field Development Plans

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ABSTRACT

A system which combines a gas reservoir simulation model with optimal development and production scheduling models has been used successfully for some time to develop long range plans for an oil and gas/NGL producing complex. The system determines how to integrate gas production from more than *40* reservoirs over a *25*-year horizon to maximize net present value (NPV) while meeting sales requirements.

The timing of drilling wells and installing compressors, as well as the number of each such facility, is affected by economic factors (taxes, royalty, price of products, cost of development and production), reservoir characteristics (permeability of producing layers, heterogeneity, conductivity of induced fractures), gas composition, gas in place and investment criteria. The criterion determining investments to add to the development schedule is *PWPI*; the incremental *PWPI* of each facility added to the optimal schedule must exceed a cutoff value of *PWPI*. The sensitivity of the optimal development schedule and desired depletion rate to the value of various parameters is presented for

1. a single reservoir with an unlimited market -- the results reveal considerable variation in sensitivity to parameters considered,
2. a single reservoir with a limited market -- the results illustrate the danger of over investing in a producing reservoir,
3. two reservoirs with a shared market -- the findings show that in some instances complex interactions between economic and reservoir parameters and market specifications give rise to an optimal strategy which does not agree with intuitive expectations – sometimes using negative cutoff values of *PWPI* results in larger values of *NPV*.

INTRODUCTION

Previously[1,2] we reported on the development and use of a software system which combines a gas reservoir simulation model with optimal development and production scheduling models. In its major application the system determines how to integrate production from a large number of oil and gas/NGL reservoirs over a *25*-year planning horizon to maximize net present value, *NPV*, while meeting gas sales requirements. The purpose of preparing this paper was to see what could be learned in a general sense about optimal gas reservoir development and production schedules through the application of this system to some typical gas reservoirs.

The Optimizing System

The optimal development model, which operates in tandem with the simulation model, determines the optimal schedule of drilling wells and installing compressors to meet a specified time profile of raw gas production rates. This schedule is built up sequentially, with additions continuing until either the specified rate is *met* in all years, or the incremental *PVR* of all facilities which can be added is less than a minimum cutoff level. (*PVR* denotes the ratio of *NPV* to present value of investment, *PWI*.) The simulation model is used in determining each possible addition's incremental *PVR*. The production rate (which may be equal to or less than the requested rate) vs *time* for the reservoir with the additional facility in place is computed; the incremental production vs time is obtained by subtracting the rate vs time profile predicted by the simulation model with the previously scheduled facilities. Incremental revenue and operating cost are then determined, and the incremental *PVR* of the possible addition is computed. On each scheduling step, of the possibilities considered, the facility with the largest *PVR* -- greater than the cutoff -- is added to the schedule

After addition of each facility, the production deficit; the difference between the scheduled and the requested production rate in each year of the planning horizon is calculated. On the next step candidates for addition are those which reduce the earliest remaining deficit and whose incremental *PVR* exceeds the cutoff. Thus, the scheduling algorithm sweeps forward through the planning horizon either meeting requested production in each year that it passes through, or leaving behind a deficit which cannot be met without violating the minimum *PVR* constraint. (In the discussion of the production scheduling model below we use the term *production vector* to denote the time vector giving the production rate and capacity in each year, and the associated *NPV* determined by the optimal development model for a specific requested profile of rates.)

Study Phase 1

The first phase of this study examined schedules generated by this optimal development model for a single reservoir when the market for gas produced is unlimited. For this market condition a very large production rate is requested in each year, and the development model determines what is the maximum economic production rate profile. The goal in this phase was to examine the impact of economic factors, reservoir characteristics and investment criteria on the optimal development schedule. Development schedules for twenty-four cases were determined.

The parameters varied in these cases were; gas price, p_g, well cost, I_w, real discount rate, i_{dr}, minimum cutoff value of *PVR*, permeability, k, skin factor, S, and gas in place, *GIP*. We found that well density and total compressor horsepower and, hence, the time to produce the reserves, are all fairly sensitive to these parameters. Because the abandonment pressure of *500 psia* was reached in all cases, total recovery is insensitive to changes in these parameters. The variations in the development plan are shown to be reasonably consistent with results of a simplified, analytical, *NPV-maximization* model. To treat situations with limited gas markets and/or more than one gas reservoir, the software system uses two production-scheduling models. Both are linear programming models. One is an overall coordination model based upon the decomposition method of mathematical programming. This coordinator considers production vectors determined by the optimal development model for a variety of requested production rate profiles for each reservoir included in a study. On each pass, the coordinator selects the weighted average mix of each reservoir's production vectors, the combination of which meets the market requirements in each year and maximizes *NPV*. The coordinator also determines the marginal value (in *NPV* units) obtained from supplying the last unit of gas in each year of the horizon.

These so-called shadow prices are used in the second production-scheduling model. This model, which considers a single reservoir at a time, determines a new requested rate profile that prospectively the coordinator can use to increase the overall *NPV*. Because the second model is based upon a linear approximation of a reservoir's production/capacity relationship, there is no assurance that its prospective rate profile is compatible with any of the previously determined development schedules for the reservoir. For this reason, the prospective rate profile is passed as a request to the optimal development model, and the corresponding economically acceptable development schedule and production vector are determined. The latter is passed to the coordinator for consideration in seeking to increase *NPV*. This overall iteration process proceeds until subsequent iterations indicate that no further improvement can be achieved.

Study Phase 2

In the second phase of this study the combined system was used to determine the optimal development schedule for the base case reservoir (*Case No. 1* in Table I) considered in Phase 1 with market rate set equal to *50 and 20 percent*, respectively, of the maximum rate determined with no constraint. The optimal strategy maintains production at or near the maximum rate for a few years, and allows production to decline thereafter. The results highlight the danger of over investing in a producing reservoir.

Study Phase 3

In the third phase of this study the combined system was used to determine optimal development schedules for a group of cases containing two gas reservoirs; a high permeability reservoir containing wet gas, and a low permeability reservoir containing dry gas with no recoverable liquids. The wet gas contained *20* percent (on a Btu basis) of recoverable liquids with a unit sales value over twice that of dry gas. As these reservoirs are supplying a common limited market, the examples explore the interplay between investment costs and sales value in determining the optimal way to meet market demand. Market constraints were;

(1) produce at the maximum uniform rate that can be supplied for *20* years, and
(2) produce at a rate equal to *150* percent of that determined in (1) for as long as is economically feasible before letting production decline to the end of the *20* year production period.

The coordinator makes either of these determinations automatically. With the first market constraint runs were made with a cutoff value of *PVR = 0.5, 0.0, -1.0 and - 10.0!* The results show this seemingly bizarre set of cutoff values to be sensible. In some instances the complex interactions between economic and reservoir parameters and market specifications give rise to an optimal strategy which does not agree with intuitive expectations. Sometimes, using negative cutoff values of *PVR* results in larger values of *NPV.*

PHASE 1 -- SINGLE RESERVOIR WITH AN UNCONSTRAINED MARKET

Phase 1 consisted of a sensitivity study to determine how the optimal development schedule for a single dry gas reservoir producing at capacity with an unconstrained market varies in response to changes in key parameters. The parameter values tested are listed in Table I. The optimal schedule was determined for the base case and each variant with real discount rates of *3 & 8 %/yr*. In each sensitivity case one of the five parameters listed was set at the variant shown giving a total of *(1 base case + 10 variants) x (2 discount rates) = 22 cases*. All of these runs were made with the cutoff value of *PVR = 0.5*; In addition the *8%* base case was run with the cutoff *PVR = 0.0 & -1.0*.

All costs, investments and prices used in these cases, which are given in Appendix A, are assumed after-tax present worth values. Furthermore, these values were unescalated in all years of the *20-year* horizon considered in the optimization calculations. This treatment implicitly assumes that all economic quantities will escalate at a common rate, so that the calculations can be carried out in *1984* dollars. This assumption is the basis for use of real discount rates, which are less than their current dollar counterparts by the inflation rate. Equations to convert after-tax values to before-tax values are given in Appendix A for some typical situations.

Table I Parameter Values Used In Phase 1 Sensitivity Studies			
	Low	Base Case	High
pgda, $ / MMBtu	1.10	2.20	4.40
S, Skin Factor	− 5	0	5
kh, md-ft.	125	250	500
Iwa, M$ / well	500	1000	2000
GIP, BCF	50	100	200
idr, %/yr	All cases run with 3 & 8 %		

Results for these *24* sensitivity cases are shown in Tables II and III. Table II gives percent recovery at the end of the shut-in year, volume of gas produced from an average well in the first year of production plus pertinent *economic* results.

Table II Results of Phase 1 Sensitivity Studies									
Case No.	Case Type	PLI, year	qgo, BCF/yr	Recv, %	Shutin year	AvgCst $/MCF	Avg PVR	ΔNPV %	Last PVR
idr = 3 % / yr									
1	Base	3.9	3.76	88.0	2003	0.27	13.7	0.0	.91
2	High kh	3.5	4.91	87.0	1999	0.23	17.7	3.2	.76
3	Low kh	4.2	2.60	87.5	2002	0.37	9.5	− 6.4	.70
4	High S	3.9	2.82	87.9	2002	0.34	10.7	− 3.8	.89
5	Low S	4.1	5.40	87.7	1997	0.24	16.1	5.0	.56
6	High p	3.4	3.72	88.3	1998	0.33	25.6	114.7	1.62
7	Low p	4.4	3.91	85.2	2003	0.24	7.3	− 54.7	.75
8	High Iwa	3.9	3.76	88.0	2003	0.35	8.2	− 4.4	.91
9	Low Iwa	3.5	3.63	88.2	2000	0.25	18.5	2.4	.67
10	High GIP	4.9	4.00	87.6	2004+	0.19	20.6	103.3	2.62
11	Low GIP	3.7	3.84	85.9	1997	0.42	8.9	− 55.0	.55
idr = 8 % / yr									
12	Base	2.8	3.46	88.0	1995	0.34	8.6	0.0	0.87
13	PWPI=0	2.8	3.46	88.7	1995	0.35	8.2	0.04	0.07
14	PWPI=-1	2.5	3.23	89.0	1992	0.40	6.7	− 0.34	− 0.06
15	High kh	2.8	4.52	87.3	1996	0.25	12.2	4.9	0.81
16	Low kh	3.6	2.46	88.0	2001	0.38	7.3	− 8.6	0.77
17	High S	3.5	2.81	88.2	2000	0.36	8.1	− 6.4	0.63
18	Low S	2.6	4.79	87.7	1995	0.25	12.4	6.3	1.34
19	High p	2.7	3.36	88.8	1994	0.37	16.8	122.2	2.54
20	Low p	3.8	3.76	85.6	2001	0.25	5.4	− 58.3	1.35
21	High Iwa	3.9	3.76	88.1	2002	0.36	6.8	− 8.7	0.52
22	Low Iwa	2.6	3.07	88.6	1994	0.28	12.6	5.3	0.74
23	High GIP	4.9	4.00	88.2	2002	0.22	14.2	92.7	0.83
24	Low GIP	2.6	3.33	85.6	1994	0.43	6.8	− 53.4	0.80

The values of q_{go}, shown in Table II, which are net of gas burned for field fuel, equal average reservoir production rate in *1986* divided by the number of wells drilled in *1985*. Thus, these values reflect deliverability tempered by the degree of depletion during the first year. The values of *PLI* shown equal G_{ps} / q_{go} divided by the number of wells drilled during *1985*. q_{go} and *PLI* were determined as explained because it was thought that the values so obtained would be most appropriate for the correlation presented in Figure 1. The length of time to produce the reserves, as indicated by the shut-in year, varies in the manner expected; for a given *GIP,* favorable reservoir or economic conditions result in earlier depletion, and conversely. The lower *GIP* leads to an earlier depletion, and the higher *GIP* to later depletion. With $i_{dr} = 3\%/yr$ depletion of the high *GIP* occurs a year beyond the end of the horizon considered. Both average gas cost and average *PVR* vary in the expected manner. The last column in Table II gives the value of incremental *PVR* computed by the model for the last facility added to the schedule (given in Table III). The fact that in most instances this value substantially exceeds the cutoff value is a result of the discrete nature of the scheduling algorithm. The column of ΔNPV measures the sensitivity of the reservoir's *NPV* to the parameter changes. Not surprisingly, *NPV* is most sensitive to gas price, p_g, and *GIP*. The magnitude of the variation caused by the other parameters is in all cases less than *10 percent*. The most significant sensitivity is that shown to changes in the cutoff *PVR*. Decreasing the cutoff *PVR* to zero increases *NPV* by only *0.04%*; allowing uneconomic decisions by lowering cutoff *PVR* to *−1.0* causes *NPV* to decrease by *−0.34%* relative to the *8%* base case. The great significance arises because the first variant causes investment relative to the base case to increase by *5.4%* and the second by *28.1%*. These data are indicative of the need to continually guard against over investing in producing reservoirs.

Table III gives the optimal schedule of drilling wells and installing compressors. Note in Table III that the bulk of development takes place in the first year, *1985*, including in every case the installation of at least one compressor. The model assumes that first production from a facility occurs in the year after installation. Thus, first production occurs in *1986* giving a *19-year* producing horizon through the end of *2004*. For all of the *16* cases in which two compressors were installed, the "*D*" in Table III denotes the year in which the pair was converted (at zero cost) to operate in tandem to provide two-stage compression. Examination of the recoveries in Table II reveals that in the four cases in which favorable reservoir conditions caused the second compressor to be uneconomic (*high kh* and *low S*), percent recovery was only slightly less than for other cases with two-stage compression. However, for the four cases in which reduced revenue caused the second compressor to be rejected (*low p* and *low GIP*), percent recovery fell by *2 − 3 percent*. Table III presents *two* somewhat surprising features.

1. For none of the reservoirs considered is it optimal to produce awhile without compression.
2. In seven of the cases it is optimal to postpone some drilling beyond 1985, in two instances as late as 1989.

Correlation of NPV-Maximizing Results

Eq.(7B) in Appendix B indicates that the optimal producing life index, *PLI* -- when, as in Phase 1 of our study, the market is not limiting and the rate of hydrocarbon production declines exponentially -- is a function only of the discount rate and the “pseudo payout time”, $I_{wa} / (m_v * q_{go})$. In deriving Eq.(7B) we assumed that the wells have uniform cost and deliverability characteristics and are all drilled at $t = 0$. To test this theoretical relation with our results, it is necessary to take into account the cost of compressors. This requirement was addressed by defining an adjusted well cost which includes the proportionate share of the cost of compressors installed in each case. Eq.(4B) which defines the adjusted well cost, I_{wae}, also includes the cost of the single separator/dehydrator and the fixed cost of a compressor installation. An argument might be raised that, since all development plans each included one of these latter two facilities, their cost is fixed and should not be included in the cost which varies with the number of wells. Such a modification in procedure would have made only a slight difference in the results obtained.

In Figure 1 our Phase 1 results are compared to the theoretical curve. Values of I_{wae} were computed for each case using the cost data given in Appendix A and the values of n_w & n_c listed in Table III. The values of $i_{dr}*I_{wae} / (m_v*q_{go})$ plotted on the abscissa in Figure 1 were calculated using

q_{go} from Table II and the values of i_{dr} and m_v for each case. The values plotted on the ordinate were obtained by multiplying the values of *PLI* given in Table II by the applicable value of i_{dr}, Although the points move in the general direction called for by the theoretical curve, agreement is not very good. Probably the factor contributing most heavily to the discrepancy is the assumption that the exponential curve adequately represents the decline characteristics of the dry gas reservoirs considered. Examination of results of our simulations of reservoir behavior indicates the decline rate to be considerably greater than is predicted by the exponential curve. As a result the optimal scheduling model finds gas wells to be poorer investments than they are represented to be in Eq.(7B), and, hence, elects to drill fewer of them. The effect of this election is to lower the reservoir's initial production rate and to increase PLI thus contributing to the discrepancy shown in Figure 1.

Table III Development Schedule for Phase 1 Sensitivity Studies

Case No	Year											Total	
	1985	86	87	88	89	90	91	94	96	97	00	Wel	Cp
idr = 3 % / yr													
1	6W,1C										1CD	6	1
2	5W,1C					D						5	1
3	8W,2C	1W				D						9	2
4	8W,1C					1C	D					8	2
5	4W,1C	2W										6	1
6	7W,2C					D						7	2
7	5W,1C											5	1
8	6W,1C										1CD	6	2
9	7W,1C								1CD			7	2
10	9W,2C							D				9	2
11	3W,1C	1W										4	1
idr = 8 % / yr													
12	9W,2C					D						9	2
13	9W,2C				1W	D						10	2
14	11W,2C	1W	1W		D							13	2
15	7W,1C											7	1
16	10W,2C					D						10	2
17	9W,2C					D						9	2
18	7W,1C											7	1
19	10W,2C	1W				D						11	2
20	6W,1C											6	1
21	6W,1C									1CD		6	2
22	11W,1C		1C			D						11	2
23	9W,2C			2W	1W	D						12	2
24	5W,1C											5	1

PHASE 2 -- SINGLE RESERVOIR WITH AN LIMITED MARKET

Two runs were made to study the impact of limited markets on the optimal development schedule for the *8%* base case reservoir of Phase 1. In these runs, here designated Case Nos. *25* and *26*, the maximum raw gas production rate before removal of field fuel was set at one-half and onefifth, respectively, of that of the base case's corresponding annual average daily rate, *90MMCF/D*. In Tables IV and V the recovery and economic statistics and the development schedules, respectively, are compared to the base case values. As expected, the optimal strategy is to maintain production at or near the maximum rate for a few years, and then let production decline for the remainder of the reservoir's life or the planning period, whichever is longer.

For the higher rate the length of the producing plateau is three years, *1986-8*. Perusal of Table V indicates that the model kept up development through *1987* to maintain the plateau rate through 1988, but then found it economic to let the reservoir decline for two years before installing the second compressor, which it elected to use for one year in first stage compression before converting to second stage at the beginning of *1992*. Interestingly enough, the plateau rate never actually equaled the maximum, falling short by *480 MCF/D* in *'86-7* and *1,114 MCF/D* in *'88*.

For the lower rate the optimal scheduling model elected to maintain the producing rate at the maximum allowed for nine years, letting production rate begin to decline in *1995*. Again, the economic strategy turned out to be to let production rate decline for a few, in this case four, years before installing the additional compressor in second stage compression. Table IV shows that curtailing production caused the average *PVR* to increase and the average cost of gas to decrease. These favorable indications notwithstanding, *NPVs* for Cases *25* and *26* were *4.3* and *25.4* percent lower, respectively, than for the unconstrained base case. With the lower rate the percent recovery is slightly less than the base case's value because production from the reservoir actually continued through *2005*.

PHASE 3 -- TWO RESERVOIRS WITH A SHARED MARKET

In Phase *3* four runs were made to study the interaction between the development and production schedules of two reservoirs which together are optimally supplying a common market. These runs -- denoted *Case Nos. 27-30* in Tables VI and VII -- used the system's optimal production scheduling model along with the optimal development model. For these cases the planning horizon was expanded by *1* year to extend through *2005*. In all four cases *NPV* was maximized with $i_{dr} = 8$ *%/yr*, but the market specification in *Cases 27-29* was greatly different from that in *Case 30*. In the first three cases the actual value of the dry gas Btu sales requirement was not specified. Rather the system was required to maintain dry gas Btu sales rate constant throughout the *20- year* producing horizon, *1986-2005*. The actual value was determined during the optimization along with the corresponding development and production plan for each reservoir. Making this overall determination automatically is one of the market options programmed into the system.

Aside from market specification, all data were identical for the four cases considered in this phase. *Cases 27-29* differed only in that cutoff *PVR = 0.5, 0.0, and −1.0*, respectively. In *Case 30* cutoff *PVR = 0.5*, but an alternative market specification was made. In all four cases the first reservoir, *R1*, was a copy of the base case considered earlier with the exception that *kh = 125 md-ft*. In the second reservoir *kh = 500 md-ft*, and the energy content of the gas was set equal to *1200 Btu/MCF*. Calculation of the volume of gas required to meet field fuel requirements was based upon this higher value. For *R2's* gas entering the flowline *200 Btus* were assumed to be liquids, recovered in the plant and sold separately, as discussed in Appendix A. The remaining *1000 Btus* were handled in exactly the same manner as the dry gas from *R1*.

In *Cases 27-29* the optimal plans are the result of a balance of two primary economic tradeoffs;
(1) Increase total development cost to increase gas recovery and cash flow from the larger number of Btus sold.
(2) Achieve higher cash flow early on from greater liquid sales by more intensive development in and higher production from *R2* with corresponding postponement of activity in *R1*.

Results show both of these tradeoffs are strongly influenced by the cutoff value of *PVR*. As shown in Table VII, total Btus sold increased by *2.3 and 3.6 percent* as *PVR* decreased from *0.5* *to 0.0* and from *0.0 to −1.0*, respectively. The preference for *R2's* gas is reflected in the fact that recovery in *R2* increased by *6.7 percent* between *Cases 27 and 29*, while recovery in *R1* increased by only *2.5 percent*. As can be seen in Table VI, as cutoff *PVR* decreases the total number of wells drilled in both reservoirs goes up from *11 to 14 to 19*, and development in *R2* tends to be earlier and in *R1* later. (Note that in evaluating a candidate for possible addition to the optimal schedule, incremental production and *PVR* are determined by simulating behavior with and without the facility in question for a period beyond the date of installation equal to the years of production in the planning horizon, here *20*.) The most striking feature about the results is the continuing increase in *NPV* as *PVR* decreases below *0.0*. The increase in *NPV* between *Cases 27 and 28* is expected, although the magnitude of the increase, *3.2 percent*, is larger than Phase 1 results would suggest. The further increase by *4.7 percent* when *PVR* changed from *0.0 to −1.0* runs counter to expectations and arises because of the twofold adjustment in the development and production schedules examined above. (An additional run was made with cutoff *PVR = −10*, but the results were so close to those of *Case 29* that we elected not to present them.

In *Case 30* the maximum dry gas sales level was set equal to a Btu rate approximately *50 percent* greater than that achieved in *Cases 27-29*. As expected the optimal strategy is similar in character to that of Phase 2; sales rate is maintained on a plateau for *10 years*, and then allowed to decline for the rest of the horizon. As shown in Table VI, the additional Btus are obtained by increasing recovery in both reservoirs, with the increase in *R1* being larger than in *R2*. However, the preference for *R2* is reflected in the fact that its peak production rate occurred near the beginning *of* the horizon, whereas peak rate from *R1* showed itself in *1996*, the first year beyond the plateau's end. Interestingly enough, the average cost of gas listed in Table VI for *Case 30* is nearly identical to that obtained in *Case 27*, which also used *PVR = 0.5*. This similarity is consistent with the fact that the development schedule is slightly more intensive and the volume of dry gas sold is greater in *Case 30* than in *Case 27*.

NOMENCLATURE

cc	= annual operating cost of a compressor, excluding field fuel used for power, $/yr.
cca	= annual after-tax operating cost of a compressor, excluding field fuel used for power, $/yr.
cga	= average after-tax cost of a reservoir's gas (See Eq.(9A)), $/MCF.
cs	= annual operating cost of a separator/dehydrator unit, $/yr.
csa	= annual after-tax operating cost of a separator/dehydrator unit, $/yr.
cw	= annual operating cost of a producing well, $/yr.
cwa	= annual after-tax operating cost of a producing well, $/yr.
cp	= trunk line and processing plant operating cost, net of plant fuel, $/MMBtu of dry gas sold.
f	= inflation rate, %/yr.
fpf	= fraction of dry inlet plant gas burned as fuel.
GIP	= initial gas in place, BCF.
Gps	= cumulative gas produced at shut in, BCF
I	= investment amount made at t = 0, $.
Ia	= after-tax present worth of I, $
Ia	= value of *Ia* adjusted for Australian royalty allowance for surface facilities, $.
Ica	= after-tax present worth cost of a compressor, $.
Ifa	= after-tax present worth cost to install a compressor.
Isa	= after-tax present worth cost of a separator/dehydrator, $.
Iwa	= after-tax present worth cost of a well, $.
Iwae	= Iwa plus allocated compressor cost (See Eq. (4B)), $.
in	= intangible fraction of an investment.
idr	= real discount rate, %/yr.
itc	= investment tax credit fraction.
kh	= permeability- thickness, md-ft.
mv	= average after-tax net revenue from gas sold, $/MCF.

NPV = net present value of a development and production plan, $.
n = straight-line depreciation life, years.
nc = number of compressors installed in a reservoir.
nw = number of producing wells drilled in a reservoir.
PLI = Producing Life Index (See Eq.(6B)), years.
PWCa = after-tax present worth operating cost of a reservoir, $.
PWIa = after-tax present worth of reservoir development cost, $.
pg = price of gas effluent the processing plant, $/MMBtu.
pgda = producer's net revenue after tax and royalty for trunk line inlet gas, $/MMBtu.
pl = price of recovered liquids, $/MMBtu.
pla = producer's net revenue after tax and royalty for trunk line inlet liquids, $/MMBtu.
pgwa = producer's net revenue after tax and royalty for trunk line inlet wet gas, $/MMBtu.
qg(t) = reservoir production rate at time t, BCF/yr.
qgo = initial production rate from a well, BCF/well/yr.
r = gross royalty, %.
ra = Australian Govt. royalty on net production revenues, %.
rs = recovery at shut in of a reservoir, %.
S = skin factor.
t = time, years.
ti = income tax rate, %.
xtc = fraction of investment tax credit removed from depreciable basis.

REFERENCES

1. Dougherty, E. L., Zagalai, B., O'Donnell, B., Goode, P. and Hollis, R., "A New Systems Approach to Optimizing Investments in Gas Production & Distribution", SPE 11292, J.Pet.Tech. (Jan 1986).
2. Dougherty, E. L., Zagalai, B., O'Donnell, B., Goode, P. and Hollis, R., "A New Faster Gas Reservoir Simulation Model", SPE 11731 J.Pet.Tech. (Nov 1985).
3. Dougherty, E. L., "Guidelines for Proper Application of Four Commonly Used Investment Criteria", SPE 13770 presented at 1985 SPE Hydrocarbon Economics and Evaluation Symposium, Dallas (March 14-15, 1985).

APPENDIX A

In these studies the quantity maximized was NPV after all taxes, royalties, and operating and investment costs. To avoid the confusion and complexity introduced by specific consideration of income tax and royalty computations, the determining economic parameters are all given on an after-tax basis. In this Appendix we discuss the parameter values used and indicate the relation between these values and the corresponding values before tax and royalty.

Before vs After-Tax Capital Costs

To relate an actual investment cost to the values used in this study, the actual values must be adjusted for the effects of an intangible deduction, an investment tax credit, depreciation procedure and income tax rate to obtain the equivalent after-tax present worth cost. With

1. income tax rate of *ti %*,
2. straight line depreciation over *n years*,
3. investment tax credit of *itc %*,
4. intangibles equal to *in %* of the actual investment cost, *I*,
5. reduction of the initial depreciable basis by *xtc* % of the investment tax credit,
6. approximating *1/2 year's* depreciation in *years 1 & n+1* by spreading depreciation deductions between *t = 1/2 & t = n+1/2*;
7. using continuous discounting at a real discount rate of *idr %/yr*, and
8. deflating depreciation deductions at *f %/yr*,

the ratio of the after-tax investment cost to the actual investment is given by

$$(1A)\quad \frac{I_a}{I} = i_n * \left(1-t_i\right)+\left(1-i_n\right)* \left\{1-i_{tc}-t_i*\left(1-x_{tc}*i_{tc}\right)*\left(\frac{\left(1-e\left(-n*\left(i_{dr}+f\right)\right)\right)*e\left(-0.5*\left(i_{dr}+f\right)\right)}{n*\left(i_{dr}+f\right)}\right)\right\}$$

Consider the following cases of Eq. (1A) with t_i = *45%*, *n = 5 years*, *xtc = 50%*, *idr = 8%/yr* and *f = 10%/yr*.

1. With *in = 100%* well cost is expensed and *Ia/I = 0.55.*
2. With *in = 0%*, *itc = 100%*, *xtc = 100%*, *Ia/I = 0 –* tax is reduced by the cost of the well.
3. With *in = 0%*, *itc = 0%*, well cost is recovered through depreciation -- *Ia/I = 0.729.*
4. With in = *0%*, *itc = 10%*, *xtc = 0%*, *Ia/I = .629.*
5. With *in = 0%*, *itc = 10%*, *xtc = 50%*, *Ia/I = 0.642.*
6. With *in = 10%*, *itc = 10%*, *xtc = 50%*, *Ia/I = 0.633.*
7. If the ACRS depreciation rate for a *5-year* property (the class into which oil and gas producing equipment fall) is used instead of straight line with *n = 5*) the result obtained is *Ia/I = 0.624.* The latter two results demonstrate the fact that with a judicious choice of *n* a good approximation to *Ia/I* can be obtained from Eq(1A) for any depreciation formula. This is an important consideration, since for unit of production depreciation the annual depreciation deductions are proportional to the production rate and cannot be determined exactly until the production schedule is known. Taking *n* equal to one-half of the producing life should yield a reasonably good approximation to *Ia/I* in this case

As another example of an after-tax adjustment, consider the royalty collected by the Australian Government on net revenues derived from hydrocarbon production. The capital costs of surface facilities are amortized against net operating revenue in computing royalty payments. The amortization period is 15 years and the formula used to calculate the annual amortization deduction allows recovery of original capital cost plus an internal rate of return on the unamortized portion of the investment equal to an official Australian bond interest rate. Denoting the latter as *ib %/yr* and the royalty rate by *ra %*, the after-tax adjustment, *Ia/I* caused by this royalty treatment is given by

$$(2A)\quad \frac{I_{ar}}{I} = r_a * i_b * \left(1-t_i\right) * \frac{\left(1-e\left(-15*\left(i_{dr}+f\right)\right)\right)}{\left(i_{dr}+f\right)*\left(1-e\left(-15*i_b\right)\right)}$$

With *ra = 10%* & *ib = 18%/yr* and the other factors as in the previous example, Eq. (2A) gives *Ia/I = .055*, indicating that royalty amortization reduces the equivalent after-tax cost of a surface facility by *5.5 %.*

After-Tax Capital Costs

The following after-tax-and-royalty facility costs were used in this study:

All coste are in US $	
Wells, Iw	$1,000,000 apiece
Compressor, If, Site Preparation	$250,000
Cmpressor, Ic, Capital Cost	$1,500,000 / 1000HP

Separator/Dehydrator, Iw	$1,200,000 / 120 MMCFD Unit
Compressor Efficiency	87.5 %
Operating Days / Year	346
Conversion Cost, $1^{st} - 2^{nd}$ stage	0

Prices of Gas and Recovered Liquids

Price of gas, *pgda*, used in this study is the net revenue received by the producer in *1984* dollars after all taxes and royalty when *1 MMBtu* of dry gas is delivered into the gathering trunk line exclusive of field operating costs. For the base case this value equaled *$2.20/MMBtu*, with the variants considered being one half and twice this amount. For a dry gas reservoir the actual dry gas price, *pg*, corresponding to a value of *pgda*, depends upon the nature and amount of the taxes and royalties paid and the unit operating costs in the trunk line and the processing plant. With a gross royalty of *r%*, an income tax rate of *ti%*, a unit operating cost of *cga*, and assuming that *fpf%* of the Btus inlet to the plant as dry gas are consumed as plant fuel, the relation between these prices is

$$(3A)\quad p_{gda} = \left[p_g * (1-r) - c_{ga}\right] * \left(1-f_{pf}\right) * \left(1-t_i\right)$$

For example if *r = 10%*, *cga = 0.12*, *ti = 45%* and *fpf = 10%*, the values of *pg* corresponding to the *pgda* used in this study are *2.62, 5.23, & 10.46 $/MMBtu*. Since it was assumed that the dry gas contained 1 MMBtu/MCF, these values are also the price in $/MCF.

The wet gas was assumed to contain 1.2 MMBtu/MCF) of which 0.2 MMBtu was recovered as liquids in the processing plant and sold at a price (after taxes and royalty) of *pla $/MMBtu*. Assuming only dry gas to be burned as plant fuel the price of wet gas inlet to the plant is given by

$$(4A)\quad p_{gwa} = (5/6) * p_{gda} + (1/6) * p_{la}$$

The relation between pgda and pg is given by Eq. (3A), and the analogue of Eq. (3A) relating pla to pl is

$$(5A)\quad p_{la} = p_l * (1-r) * \left(1-t_i\right)$$

In this study we set *pla = $4.50/MMBtu*. With the parameters given above Eq. (5A) gives pl = $9.09/MMBtu, which in retrospect is an unrealistically high value. (Another way of looking at this value is to conclude that the liquids content of our wet gas is understated.)

Operating Costs

Field operating costs were determined for each case from the following after-tax values of annual operating cost of facilities:

ca (separator / dehydrator)	250,000 $ / yr
ca (compressor)	250,000 $ / yr
cwa (well)	25,000 $ / yr

In Australia surface, but not downhole, operating costs are deducted before computing royalty on net revenue. With this stipulation actual costs are obtained from the tabulated values using the following formulas

(6A) csa = cs*(1 − ra)*(1 − ti)

(7A) cca = cc*(1 − ra)*(1 − ti)

(6A) cwa = cw*(1 − ti)

Field fuel requirements for the separator/dehydrator and for each compressor unit are input in MMBtu/day (operating). Using these values the optimal field development model determines the fraction of the raw gas produced which is burned to power the field facilities. In making this determination the actual compressor horsepower required in each year is computed, and the compressor fuel requirements are based upon this latter value. For the base case volume of gas actually burned decreased by about *25%* from the peak production year to the year of shut in. On the other hand, the percentage of the raw gas stream burned in the field increased steadily from less than *1%* in the first year of production to nearly 20% in the shut in year. This profile of field fuel consumption is typical of all cases considered.

The raw gas was assumed to contain *7.42%* of inerts. The presence of these diluents increased the cost of field facilities required to supply a specified Btu off take, but the rate of increase is not a direct proportion. Hence a simple formula for scaling this parameter cannot be given. The unit cost of gas defined by the following equation provides a useful measure of development and production costs

$$(9A)\ PWI_a + PWC_a = c_{ga} * \int_0^{21} q_g(t) * \exp(-d_r * t)\, dt$$

cga is the average after tax cost, *\$/MCF*, which uniformly spreads the present worth of total after tax production and development costs over the gas produced during the horizon. In these studies *qg(1) = 0* because initial development was assumed to take place in *year 1*, with first production starting at the beginning of *year 2*. Values of *cga* for all reservoirs in all cases are included in the tables of results. Note that the q*g* used in Eq. (9A) is the gas flow rate into the trunk line after removal of field fuel. Thus, *cga* includes the actual cost of supplying field fuel.

APPENDIX B

NPV Maximization for a Single Reservoir

Assume that production rate declines exponentially. Then with n_w wells gas production rate is given by

$$(1B)\ q_g(t) = n_w * q_{go} * \exp(-n_w * D * t)$$

where, *D*, the decline rate for a single well, is given by

$$(2B)\ D = q_{go} / G_{ps} = q_{go} / [r_s * GIP]$$

Here we have denoted gas recovery at shut in by $r_s = G_{Ps}/G$. If the reservoir is produced at capacity at all times, NPV is given by

$$(3B)\ NPV = -I_{wa} - I_{ca} + \int_0^{\infty} m_v n_w q_{go} \exp(-[n_w D + i_{dr}]\, t)\, dt$$

$$(3B)\ NPV = -n_w I_{wae} + \frac{m_v n_w q_{go}}{n_w D + i_{dr}}$$

where the after-tax marginal revenue, mv, has been assumed to be constant. The "effective" well cost is obtained by spreading the cost of compressors, Ica, over the nw wells, viz.

$$(4B)\ I_{wae} = [n_w I_{wa} + n_c I_{ca} + I_{fa} + I_{sa}] / n_w$$

Differentiating Eq. (3B) with respect to nw, neglecting the dependence of $Iwae$ on nw, setting the result equal to zero, after rearranging and substituting Eq. (2B), the following expression is obtained for the optimal number of wells, nwo:

$$(5B)\ [i_{dr} / (n_{wo} + i_{dr})]^2 = I_{wae} / [m_v q_{go}]$$

Based upon the NPV-maximizing initial rate, $nwo*qgo$, the producing life index, PLI, is given by

$$(6B)\ PLI = G_{ps} / [n_{wo} q_{go}] = 1 / [n_{wo} D]$$

Introducing Eq. (6B) into Eq. (5B) and solving for PLI gives the following:

$$(7B)\ i_{dr} * PLI = \frac{[i_{dr} * I_{wae} / (m_v q_{go})]^{1/2}}{1 - [i_{dr} * I_{wae} / (m_v q_{go})]^{1/2}}$$

Note that the lhs and the term in brackets an the rhs are dimensionless. Eq. (7B) states that the value of $idr*PLI$ that maximizes NPV is a function only of the product of idr times the 'pseudo payout time', $Iwae / (mv*qgo)$. The latter is the length of time in years required to pay out the effective well cost, $Iwae$, if the well were to produce continuously at its initial rate, qgo. Eq.(7B) provides the basis for correlating Phase 1 results by plotting idr * PLI vs $idr*Iwae / (mv*qgo)$ in Figure 1.

Table IV Phase 2 Results – Single Reservoir with Limited Markets									
Case No.	Case Type	PLI, year	qgo, BCF/yr	Recv, %	Shutin year	AvgCst $/MCF	Avg PWPI	ΔNPV %	Last PWPI
idr = 8 % / yr									
12	Base	2.8	3.46	88.0	1995	0.34	8.6	0.0	0.87
25	45MMCF	5.8	3.82	88.4	1997	0.30	9.9	– 4.3	1.13
26	18MMCF	14.3	3.06	87.2	2004+	0.27	11.7	– 25.4	0.52

Table V Phase 2 --Development Schedule – Single Reservoir – Limited Markets												
Case No	Year										Total	
	1985	86	87	88	89	90	91	92	93	94	99	Wel	Cp
idr = 8 % / yr													
12	9W,2C					D						9	2
25	4W	2W	2W,1C			1C	D					8	2
26	2W			1C	1W			1W	1W	1W	1CD	6	2

Table VI Phase 3 Results – Two Reservoirs with a Shared Market									
Case No.	Case Type	PLI, year	qgo, BCF/yr	Recv, %	Shutin year	AvgCst $/MCF	Avg PWPI	ΔNPV %	Last PWPI
idr = 8 % / yr									
27R1	Cutoff	28.9	2.75	72.3	2005+	0.33	9.1	0.0	0.57
27R2	PWPI=0.5	25.3	5.63	78.1	2005+	0.22	22.1		1.07
28R1	Cutoff	37.1	2.77	73.8	2005+	0.34	8.5	3.2	0.11
28R2	PWPI=0.0	20.4	5.63	80.6	2005+	0.22	22.1		0.85
29R1	Cutoff	96.4	2.77	74.8	2005+	0.40	6.6	7.9	−0.07
29R2	PWPI=−1.	14.6	5.41	84.8	2005+	0.25	19.4		−0.63
30R1	9.9	19.8	2.75	83.1	2005+	0.33	8.9	30.9	0.70
30R2	TBtu/yr	16.6	5.41	86.6	2005+	0.21	23.4		0.58

Table VII Phase 3 --Development Schedule – Two Reservoirs – Shared Market								
	Case No. 27		Case No. 28		Case No. 29		Case No. 30	
	Reservoir							
Year	R1	R2	R1	R2	R1	R2	R1	R2
1985	2W	1W	1W	1W	1W	2W	2W	2W
1986			1W					
1987								
1988								
1989						1C	1W	
1990		1W		1W	1W		1W	
1991	1C		1W			1W	1C	1C
1992					1W			1W
1993			1C				1W	
1994	1W						2W	
1995					1C		1W,1C	1W
1996		1C	1W		1W			
1997	1W			1C			D	1W
1998				1W	2W			
1999								
2000	1W	1W	1W			1W		
2001	1W		1W,1CD		1W,1CD			
2002	1CD	1W	1W	1W		1W,1CD		
2003	1W	1CD	1W	1W	1W	1W		
2004			1W	1CD	5W			
2005								
Total	7W,2C	4W,2C	9W,2C	5W,2C	13W,2C	6W,2C	8W,2C	5W,1C
DryGas Sold	131.5 TBtu		134.5 TBtu		139.4 TBtu		198.0 TBtu	

TBtu = Trillion Btu

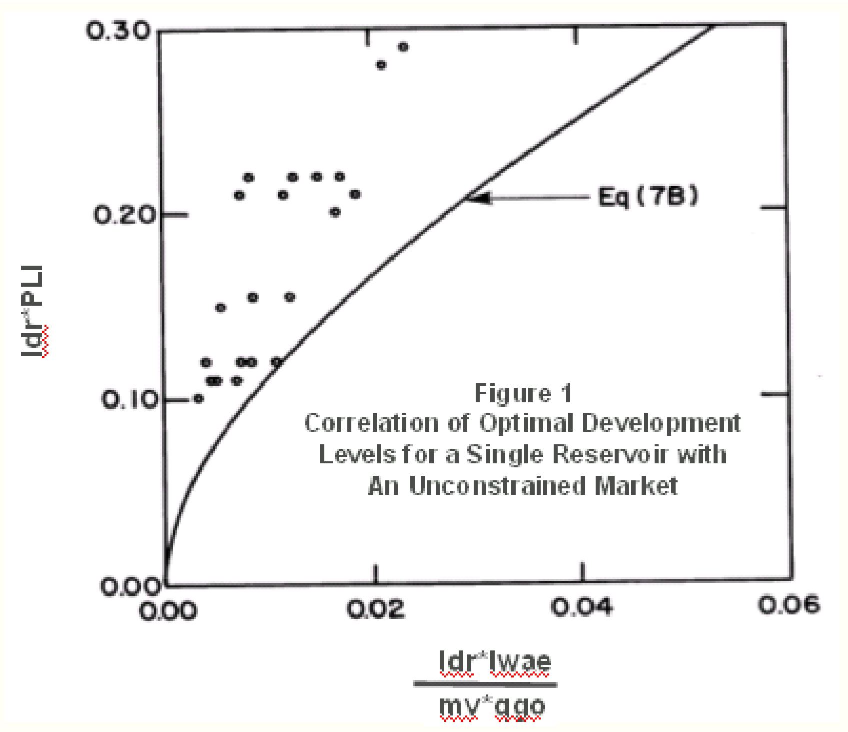

Figure 1
Correlation of Optimal Development Levels for a Single Reservoir with An Unconstrained Market

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