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Guidelines for Proper Application of Four Commonly Used Investment Criteria

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ABSTRACT

Two aspects of the most favored investment criterion, discounted-cash-flow rate of return, $\tilde{R}$, (*DCFROR*), that continue to confound evaluation engineers are:

1. '*How to interpret the value obtained*', and
2. '*What to do when dual values arise*'.

Using continuous discounting formulas, we demonstrate mathematically that when a sing1e value is obtained, $\tilde{R}$ is the rate of return realized on the ***un-amortized*** investment.

Three alternative investment criteria used to cover the breach caused by the occurrence of dual values of $\tilde{R}$ are:

1. *PVR*, ratio of net present value to present worth of investment always is consistent.
2. $\tilde{G}$ *(GROR)*, growth rate of return, can be computed directly from *PVR*, so is always satisfactory.
   - As we demonstrate, with these two criteria care must be exercised to specify the investment properly.
3. Modified *DCFROR*, R_m
   - Because a standard procedure for computing it cannot be defined for all projects, R_m is not a consistent economic evaluation yardstick.

Finally, we show the proper way to escalate prices and costs in evaluations. With depreciation treated correctly, escalated *DCFROR*, $\tilde{R}c$, equals real

DCFROR, $\tilde{R}r$ *,* plus inflation rate and both procedures yield the same value of *PVR.*

INTRODUCTION

Next to payout, $\tilde{R}$ is probably the economic yardstick most universally perceived as providing a meaningful measure of the desirability of a proposed investment. $\tilde{R}$ is an earnings rate whose value measures the efficiency with which an investment project uses the funds invested. $\tilde{R}$ can be compared with other possible dispositions of the invested funds, such as the interest rate paid by money market accounts or on U.S. Treasury securities, thus measuring the efficiency of a project's earnings power relative to a relatively risk-free norm. Unfortunately, $\tilde{R}$ behaves in an aberrant manner in situations that arise frequently in the oil patch, and we must turn to other measures to deal with these occurrences. Like an old and trusted friend with moderate human frailties, we carry $\tilde{R}$ along seeking its counsel when it is behaving well and looking the other way when it is "acting up."

Given its actual shortcomings, it is unfortunate that uninformed rumor charges our old friend with a serious fault that it does not have. This misrepresentation is, "***A basic weakness of DCFROR is that, for an investment to produce the stated earnings rate, the cash flows from the project must be reinvested at the project's rate of return***." Although Stermole[(1)] pointed out the fallaciousness of this statement three decades ago, the misconception lingers on. Here, we exonerate our friend, proving mathematically that $\tilde{R}$ ***is the earnings rate on the unamortized (unrecovered) portion of the investment***. In preparation for this proof, we first briefly review the formulas for continuous discounting.

The fundamental aberration of the *DCFROR* technique is giving two different values for certain classes of problems. Here, we sketch how to recognize from a project's cash flow if dual values of $\tilde{R}$ will occur. In general, most organizations calculate either *PVR* or G as a companion to $\tilde{R}$ for all investment projects and use the alternate as key decision criteria when dual values of $\tilde{R}$ occur. We demonstrate that either *PVR* or G provides a uniformly consistent measure of investment performance in all cases, ***provided the project's investment is properly computed***. Since a simple analytical formula relates *PVR* and G, consistency of one implies consistency of the other.

Modified *DCFROR* was proposed as a third yardstick to fill the dual-rate gap. We show by example that a smooth transition from $\tilde{R}$ to modified R_m does not occur. Thus, R_m must be used for all projects, whether they yield dual values of $\tilde{R}$ or not, if consistent measures are to be obtained across the board. Blanket use of the modified procedure is not convenient or straightforward, so modified *DCFROR* is not a viable alternate economic yardstick.

Rapid increase in oil and gas prices, followed by a similar run-up in costs, and then a price downturn has caused analysts often to escalate prices and costs in evaluations. These inflated numbers may

1. *be used directly to calculate economic yardsticks, or*
2. *be deflated and the resulting values used. In this case the depreciation deductions are also deflated.*

With the 1st option current dollar rates of return, $\tilde{R}c$ and $\tilde{G}c$, are obtained, whereas the 2nd yields the real dollar rates of return, $\tilde{R}r$ and $\tilde{G}r$. Each real value is less than its inflated counterpart by the rate of inflation. If performed properly, both procedures yield the same value of *PVR*.

CONTINUOUS DISCOUNTING PROCEDURES

Discounting procedures measure the economic equivalence between net cash flows occurring at different points in time by taking into account the interest which funds invested at interest rate, ***i***, earn during the time interval considered. These equations differ in form depending upon the frequency with which interest earned is compounded into interest-bearing principal. We assume throughout that interest is compounded continuously. Since all readers may not be familiar with the formulas for continuous discounting, we summarize them here. In Figure 1, which depicts the time profile of cash flows to which these formulas apply, nomenclature similar to that of Stermole is used.

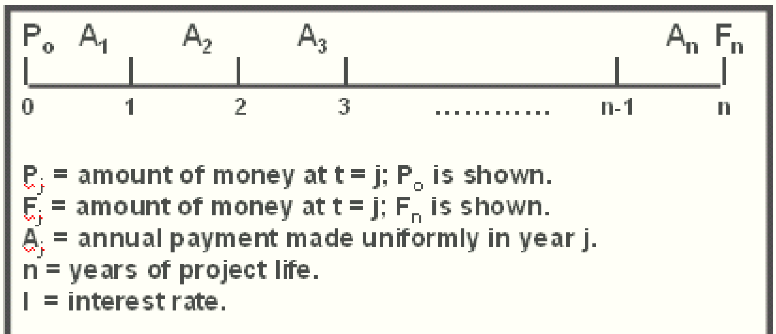

Figure 1 Time Diagram For An Investment

Continuous discounting formulas can be derived starting with compound interest equations for *n* periods and taking the limit as *n* goes to infinity. However, we obtain the desired result more readily from the differential equation that describes continuous compounding of interest. Eq.(1) states that the rate of increase in future worth equals the product of the interest rate, *i*, and the instantaneous value of (future) worth, *F,* at time *t*.

$$(1)\ dF/dt = i*F$$

The solution to Eq.1 giving, *F*, at any time *t* is

$$(2)\ F = P_o * \exp\{it\}$$

Multiplying Eq.(2) by *exp(− it)* gives the present worth of a finite cash flow occurring at a specific time, *t*, i.e.,

$$(3)\ P_o = F * \exp\{-it\}$$

so that the discount factor for such a transaction is *exp(− it).*

If payments are received uniformly and continuously at rate *A* $/yr, the rate of increase of future worth is given by

$$(4)\ dF/dt = i*F + A$$

The solution to Eq.(4), which gives the future worth of a uniform and continuous payment of magnitude *A* from *t = 0* to *t = t*, is

$$(5)\ F = \frac{A}{i} * [\exp\{it\} - 1]$$

From Eq.(3) it follows that the present worth of this payment is

$$(6)\ P_o = F * \exp\{-it\} = \frac{A}{i} * [1 - \exp\{-it\}]$$

Table I compares results obtained using continuous discounting to those gotten using other formulas. The present worth of *$100* received during a single year is given for a variety of assumptions about the timing of the payment and the method of discounting. The values shown in Table I demonstrate three noteworthy points:

1. *Continuous discounting with uniform and continuous payments gives nearly the same value as midyear discounting. The latter procedure has long been the generally accepted procedure in the oil and gas industry. (For midyear discounting the annual cash flow is assumed to occur at the middle of the year and the factor for discounting one-half year, [1 / (1 + i / 2)], is replaced by its close approximation, [1 / (i +1)**(1/2)], where the symbol ** denotes exponentiation.)*
2. *As the number of compounding periods increases, the value obtained with uniform, discrete payments converges to that obtained with continuous discounting of uniform and continuous payments.*
3. *With a single annual payment at yearend, continuous or single-period discounting give nearly the same result, a value that deviates signifi-*

cantly from that gotten with more realistic assumptions about the timing of the payment.

Table I Comparison of Computed Values of Present Worth of Total Payment of \$100 Discounted at 10% / year

CS	Po,\$	Payment Timing	Discount Method	Formula
1	95.16	Uniform & Continuous	Continuous -- Eq 6	(100/0.1)*(1- exp{-.1})
2	95.35	Single Midyear	Midyear	100 / $(1.1)^{1/2}$
3	90.48	Single Year End	Continuous – Eq 3	100 * exp{-.1}
4	90.91	Single Year End	Compound Annual	100 / 1.1
5	92.97	Equal Semiannual	Compound Semiannual	50*($x+x^2$) x=1/(1+.1/2) = 1/1.05
6	94.05	Equal Quarterly	Compound Quarterly	25*($x+x^2+x^3+x^4$) x=1/(1+.1/4) = 1/1.025
7	94.78	Equal Monthly	Compound Monthly	100/12*($x+x^2+\ldots x_{11}+x^{12}$) x=1/(1+.1/12)=1/1.0083
8	95.15	Equal Daily	Compound Daily	10/365*x($1-x^{365}$)/(1-x) x=1/(1+.1/365)=0.99973

Note with continuous discounting that if the *\$100* is received as a single lump sum payment at year-end (Case 3), the present worth is approximately *5* percent *(\$4.68 = 95.16 - 90.48)* less than if the *\$100* cash flow is received at a uniform rate throughout the year (Case 1). With income distributed, money received can be put to work earning interest during the year of receipt, whereas with the year-end lump sum payment, it cannot.

For Cases 3 & 4 the *\$100* is received as a single payment at year-end. Present worth with annual compounding (Case 4), is larger than with continuous compounding (Case 3) *(\$90.91 > \$90.48)*. This is because with simple interest (annual compounding) a larger initial balance is required for principal plus year-end interest to total *\$100* than is required when the interest on the growing balance is compounded continuously throughout the year.

Formulas for present worth can be transformed into equations for future worth using Eq. (3). Therefore, in the following we give only formulas for present worth. The formulas giving present worth

Figure 2 Time Diagram For Example 1

when the cash flow varies during the project lifetime use the numbers in the example shown in Figure 2.

The present value, P_o, of this cash flow stream with discount rate, $i_d = 12\%$ is given by

$$(7)\ P_O = \frac{100}{0.12}[1-\exp\{-0.12*2\}] + \frac{500}{0.12}[1-\exp\{-0.12*5\}]*\exp\{-0.12*2\} = 1427$$

Note the second term in this equation -- to obtain the present value of a continuous cash flow stream that does not begin at $t = 0$, the equivalent value at the start of the cash flow is computed, and this quantity is discounted back to $t = 0$ as if it were a lump sum payment.

DCFROR AND PRESENT WORTH EQUATION

Usually in an investment analysis we are interested in the net present value, *NPV*, which is the difference between the present values of the cash flows and the investment. With a single investment made at $t = 0$, denoted I_o, *NPV* is given by

$$(8)\ NPV = -I_O + P_O$$

Regardless of the discounting method $\tilde{R}$ is defined to be the discount rate which causes *NPV* of an investment project to equal zero. The value obtained; however, does depend upon the discounting method, but this variation need not concern us as long as the same discounting procedure is used with all projects.

If we calculate $\tilde{R}$ of each of a group of projects, and then rank them in order of decreasing $\tilde{R}$, the relative ranking will be the same regardless of the discounting procedure used. (We have not proven this hypothesis, but we are convinced that anomalies would only occur with projects that are effectively economically equivalent.)

With *DCFROR* the basic decision rule is:

- ***If $\tilde{R} > i_m$, the investment is acceptable; if $\tilde{R} < i_m$, the investment is unacceptable; where i_m denotes the investor's minimum targeted rate of return.***

Obviously, the condition for $\tilde{R}$ mirrors the requirement that $NPV(i_m) > 0$ for an acceptable project. Except for a few projects at the dividing line, projects that are acceptable with one discounting procedure will also be acceptable with another. As an example of a fringe project from Table I, if we were to pay *$90.60* today for a *$100* dollar payment to be received one year from now, using continuous discounting $\tilde{R}$ = *9.87%*; whereas with annual compounding $\tilde{R}$ = *10.38%*. If $i_m = 10\%$, the project would be acceptable in the second case and unacceptable in the first. The existence of such fringe projects does not concern us, because we are ambivalent to accepting or rejecting investments that are so marginal relative to our limiting criterion. Thus, we believe that the investment decisions made using continuous discounting will be economically equivalent, if not identical, to those made using one of the other forms of discounting listed above.

Using $\tilde{R}$ to select investment projects has a surprising wrinkle! *Given a group of acceptable projects, picking the project (or projects) with the largest* $\tilde{R}$ *is not the correct decision rule.* Rather, $\tilde{R}$ on incremental investment must be calculated to obtain the proper selection. The project (or projects) selected will always have the largest *NPV*. If any, or all, of a group of projects can be selected, (subject to the limit of available funds) *PVR* and $\tilde{G}$ will correctly rank the projects in order of relative desirability. This capability is another reason for preferring these latter two economic yardsticks to *DCFROR*.

MEANING OF DCFROR

Historically; it has often been said that $\tilde{R}$ has a shortcoming -- *to realize the computed value requires that net cash flow from the project be reinvested at the project* $\tilde{R}$. Even some fairly recent publications (2,3) make this statement. The statement is wrong. The correct statement is that the project yields a return equal to computed $\tilde{R}$ on the *unamortized* portion of the investment regardless of what is done with the net cash flow coming out of the project

We use the project's cumulative cash flow, *CCF(t)*, to demonstrate mathematically the correctness of this latter statement. Cumulative cash flow is the amount of money tied up in (unrecovered from) a project at any point in its life. As a simple example of *CCF(t)* consider the following case in which no interest is earned, i.e., $i = 0$. Suppose an investor "invests" *$1000* in a project that yields a uniform net cash flow of *$500/yr* for two years. Figure 3 graphs cash flow for this

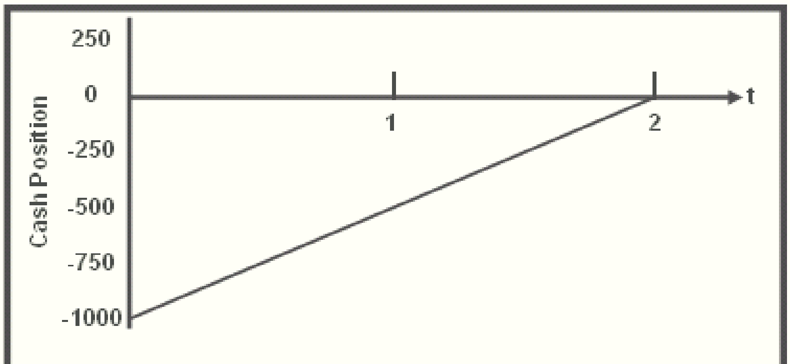

Figure 3 Cumulative Cash Position for Example 2

project vs time.

Clearly, at the end of the project, the initial investment has been paid out and no interest has been received on the unrecovered amount while it was tied up in the project. As Figure 3 shows, at $t = 0$, $CCF(0) = -I, = -\$1000$ and at $t = 2$, $CCF(2) = 0$. Mathematically, for this case we can write

$$(9)\ dCCF/dt = A;\ CCF(0) = -I_o = -\$1000$$

with $A = \$500/yr$. The solution to Eq.(9) is

$$(10)\ CCF(t) = CCF(0) + A*t = -\$1000 + 500*t$$

from which

$$(10a)\ CCF(2) = -\$1000 + 500*2 = 0$$

Eq.(10) shows that for this simple illustration $CCF(t)$ is the unamortized (unrecovered) portion of the investment at any time t. A graph of $CCF(t)$ is a straight line for $0 < t < 2$, as Figure 3 shows. At $t = 1$, $CCF(1) = -500$, confirming the obvious result that at the end of the first year the investor has gotten back half of his initial investment with no interest.

In the general case in which interest is received on the unamortized portion of investment, Eq.(9) is replaced by

$$(11)\ dCCF/dt = A + i*CCF(t)\ ;\ CCF(0) = -I_o$$

Note that initially $i*CCF(t) < 0$, since $CCF(t) < 0$. If $A > i*CCF(t)$, the right hand side of this differential equation is positive, and the rate of income is sufficient to increase $CCF(t)$, i.e., reduce the unamortized balance of the investment, while paying interest at rate i on this balance.

The solution to Eq.(11) is

$$(12)\ CCF(t) = -I_o * \exp\{it\} + [A/i]*[\exp\{it\} - 1]$$

From Eqs.(2) and (5) we see that the terms on the right hand side of Eq.(12) are the future value of I_0 and of cumulative income $(A*t)$ at time t, respectively. By Eq.(3) multiplying $CCF(t)$ by $exp(-it)$ gives its present worth. Performing this discounting operation for $t = n$, the life of the project, we obtain

$$(13)\ CCF(n) * \exp\{-in\} = -I_o + [A/i]*[1 - \exp\{-in\}]$$

Examination of Eqs.(6) and (8) reveals that the right hand side of Eq.(13) is equal to the NPV of a project with initial investment I_o and continuous cash flow at rate $A\$/yr$ from $t = 0$ to $t = n$. Thus, setting Eq.(13) equal to zero, requires that $NPV = 0$. But we saw earlier that the value of i which satisfies this requirement is defined to be the $DCFROR$, $\tilde{R}$. Hence, we can write

$$(14)\quad NPV = -I_O + [A/\tilde{R}\ ]*[1 - \exp\{-\tilde{R}*n\}] = 0$$

This completes our proof of the correct interpretation of the meaning of *DCFROR*. Eqs.(13) and (14) show that $\tilde{R}$ is the interest rate for which $CCF(n) = 0$, and Eq.(11) shows that interest is earned only on $CCF(t)$, the unamortized portion of the investment at time $t < n$.

DUAL VALUES OF DCFROR, GROWTH RATE OF RETURN & PVR

For most investment projects net cash flow is negative for one or more years, after which it becomes positive and remains so for the rest of the project's life. If, instead, after being negative and then positive for several years, the net cash flow again becomes negative for the remainder of the project, solution of the present worth equation may yield two positive values of $\tilde{R}$. In this case $\tilde{R}$ is useless as a decision criterion. This phenomenon concerns us, because oil field projects with this sort of net cash flow profile occur from time to time.

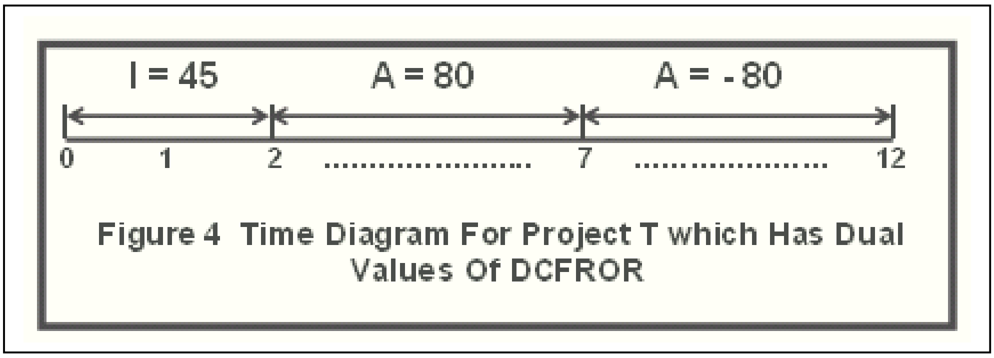

Figure 4 Time Diagram For Project T which Has Dual Values Of DCFROR

Figure 4 shows a time diagram for Project T, which yields two values of $\tilde{R}$. Note the pattern of signs of the (net) cash flows for this project. During the first two years the investment, *I*, gives a negative cash flow; in years 3-7 cash flow is positive and in years 7-12 cash flow is negative again. For this project we have

$$(15)\quad NPV(i) = -\frac{45}{i}*[1-\exp\{-2i\}] + \frac{80}{i}[1-\exp\{-5i\}]*\exp\{-2i\}$$
$$-\frac{80}{i}[1-\exp\{-5i\}]*\exp\{-7i\} = 0$$

Substituting various values of *i* into Eq.(15), reveals that $NPV = 0$ for $i = 7.1\%$ and $i = 43.8\%$.

Curve T in Figure 5 depicts the relation between *NPV* and *i* for Project T. Since NPV is positive for $7.1\% \le i \le 43.8\%$, Project T is acceptable if the investor's i_m falls within this range. In the discussion that follows we assume $i_m = 15\%$. At this discount rate Eq.(15) gives $NPV = 32.2$ so that Project T is an acceptable investment.

Figure 5 also shows plots of NPV vs i_d for three projects whose cash flows differ from that of Project T in the manner shown in Table II. The time pattern of signs shown in Figure 5 characterizes the shape of each curve. Curve N corresponds to a "normal" investment project in which the negative cash flows during the investment period are followed by positive cash flows throughout the remainder of the project's life. Such a project possesses a unique $\tilde{R}$ (in this case *24.3%*) and is acceptable whenever $i_m < \tilde{R}$, because then $NPV(i_m) > 0$. At an interest rate of *24.3%*, $CCF = 0$ at $t = 7$, so that $\tilde{R}$ is the interest rate on the unamortized investment. For curves T, U, & V the sign pattern is (-), (+), (-). We have already seen that Project T yields meaningless, dual values of $\tilde{R}$, and is acceptable only if i_m falls between these two limits. Project U has a unique positive value of $\tilde{R}$ = *45.1%*, and the project's $NPV > 0$ for $i_m < 45.1\%$. In spite of this project's seemingly normal behavior, the meaning of its DCFROR is blurred.

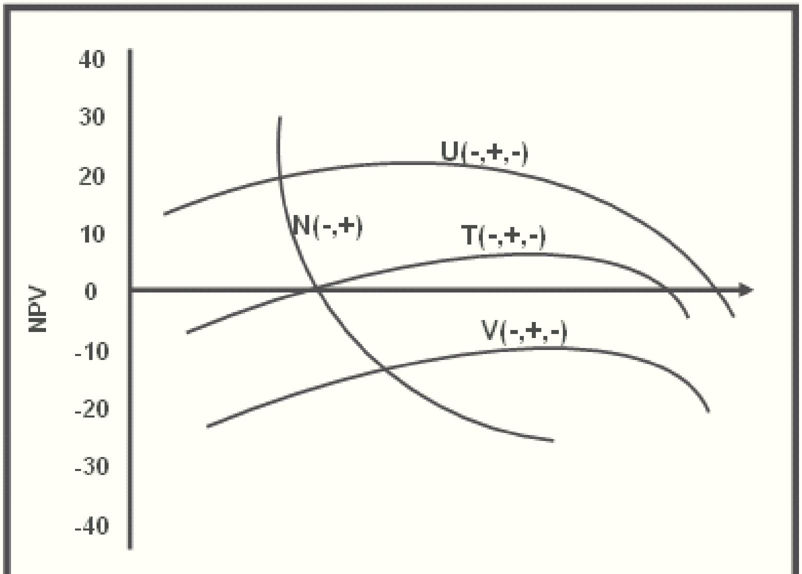

Figure 5 Types Of Relations Between NPV & i

Table II Cash Flows for Projects N, U & V			
Cash Flow, $ / yr, in Years			
Project	1 - 2	3 - 7	8 - 12
N	- 45	40	0
U	- 45	80	- 60
V	- 50	80	- 130

During the early years, the project's CCF is negative, and $\tilde{R}$ is the rate of interest earned on the unamortized investment. At a rate of interest of *45.1%*, payout of the initial investment occurs at $t = 5.85$ years; at this time $CCF = 0$. CCF increases to a maximum value of *120.6* at $t = 7$, and then decreases to zero at $t = 12$. Hence, for the *6.15* years after payout the calculation of $\tilde{R}$ implies that the cash flows are reinvested at *45.1%*, which is the weakness we wish to circumvent, Thus, we conclude that $\tilde{R}$ is not a valid investment criterion for a project with a type U sign profile. For Project V the maximum value of $NPV = -2.70$ for $i = 30\%$. This project yields no $\tilde{R}$ and is unacceptable for all discount rates.

GROR AND PVR

Growth rate of return, $\tilde{G}$, and present value ratio, *PVR*, are two investment criteria frequently used to fill the breach left by dual values of $\tilde{R}$. Since both $\tilde{G}$ and *PVR* provide satisfactory measures for decision-making on all investment projects, either or both could completely replace $\tilde{R}$. This is seldom, if ever, done because most decision makers intuitively like $\tilde{R}$.

GROR

$\tilde{G}$ is obtained in two steps:

1. *Compute future value of all cash flows, F_n, at an arbitrary point in time, $t = n$, obtained by reinvesting all cash flows from the project at interest rate i_m.*
2. *Calculate the rate of interest that PWI would have to earn from $t = 0$ to $t = n$ to achieve F_n. This value is $\tilde{G}$.*

For Project T with $i_m = 15\%$ *PWI* is

$$(16)\quad PWI = \frac{45}{0.15}\,[1 - \exp\{-0.15*2\}] = 77.75$$

The future value of the cash flows, F_n, at $t = n$ is given by

$$(17)\quad F_n = \frac{80}{0.15} * [\exp\{0.15*5\} - 1] * \exp\{0.15*(n-7)\}$$

$$- \frac{80}{0.15} * [\exp\{0.15*5\} - 1] * \exp\{0.15*(n-12)\}$$

The defining equation for $\tilde{G}$ is either of the following equivalent forms;

$$(18)\quad PWI*\exp\{n*\tilde{G}(n)\} = F_n$$

$$(19)\quad \tilde{G}(n) = \frac{1}{n} * \ln\left[\frac{F_n}{PWI}\right]$$

Here we call attention to the dependence of $\tilde{G}$ on n. For convenience, later we elect not to denote this relationship. Listed in Table III are values of F_n and $\tilde{G}(n)$ obtained from Eqs.(16)-(19) for Project T for $n = 10, 12$ & 14. This tabulation illustrates the major objection raised against $\tilde{G}(n)$ as a decision criterion; the value obtained depends upon the value of n used in the calculation. Be aware that the value of n used is arbitrary; it can be less than, equal to, or greater than the life of any individual project. This flexibility is essential, since a common value of n must be used for all projects in order to obtain consistent and comparable values of $\tilde{G}(n)$.

The variation of $\tilde{G}$ with n is always in the direction illustrated in Table III; as n increases, $\tilde{G}$ asymptotically approaches i_m. This trending towards i_m, which occurs from above or below, results because $\tilde{G}$ is a weighted average of an intrinsic rate of return on the money invested in a project and the rate of return, i_m, at which the cash flows from the project are assumed to be reinvested. The decision criterion for $\tilde{G}$ has the same form as for $\tilde{R}$:

- ***If $\tilde{G} > i_m$, the investment is acceptable. If $\tilde{G} < i_m$, the investment is unacceptable.***

The rationale for this rule becomes clear from comparison of the second and third columns in Table III. The second column gives the future worth achieved at year n if *PWI* is invested at $t = 0$ at interest rate i_m, whereas the third column gives the corresponding future worth if this same amount of money is invested in the project and its cash flows are reinvested at i_m. Whenever the latter value exceeds the former, $\tilde{G} > i_m$, and vice versa.

Table III Growth Rate of Return & Future Worth for Project T
$n = 10, 12$ & 14, $i_m = 15\%$

n	F_n of PWI=77.75	Project T F_n	GROR(n)
10	348.5	493.0	18.5%
12	470.4	665.4	17.9%
14	634.9	898.2	17.5%

PVR

Present value ratio, *PVR*, is obtained by dividing *NPV* by *PWI*, i.e.,

$$(20)\quad PVR(i) = NPV(i) \,/\, PWI(i)$$

Hence, for Project T with $i = i_m = 0.15$,

$$(21)\quad PVR\,(0.15) = 32.2 \,/\, 77.75 = 0.41$$

Since *PVR* is directly proportional to *NPV*, if $NPV(i_m) > 0$ then $PVR(i_m) > 0$, and vice versa. Thus, the decision rule for *PVR* is the same as for *NPV*:

- ***To be a candidate for selection*** $PVR(i_m) > 0$.

Here *PVR* indicates Project T to be acceptable, in conformance with the decision reached with *NPV* and $\tilde{G}$.

Consideration of Eq.(20) tells us that *NPV* and *PVR* will always give the same signal. Can we conclude that *GROR* will also always give the same signal? Fortunately, the answer is yes. The following formula relates $\tilde{G}$ and *PVR*:

$$(22)\quad GROR = \frac{1}{n} * \ln\left[PVR\,(i_m) + 1\right]$$

If $PVR(i_m) > 0$, $\tilde{G} > i_m$, and if $PVR(i_m) < 0$, $\tilde{G} < i_m$. Thus, these two economic yardsticks give a consistent result. Eq.(22) shows that $\tilde{G}$ is a combination of the earnings rate of the reinvested cash flows from the project, i_m, and the earnings rate of the project, represented by the second term on the rhs. The presence of *n* in the denominator causes $\tilde{G}$ to trend toward i_m as *n* increases. Eq.(22) indicates that if a common value of *n* is used to calculate $\tilde{G}$ for all projects, the signal generated will always be consistent with that obtained from either *NPV* or *PVR*.

Briefly, the important rules for selecting investment projects using $\tilde{G}$, *PVR* and *NPV* are:

1. ***With unlimited investment funds pick all projects with*** $NPV(i_m) > 0$.
2. ***With limited funds pick the feasible group yielding the largest total value of NPV. Except for some shuffling that may be necessary to invest the last increment of capital, this group can be identified by ranking projects in decreasing order of GROR or PVR.***

MODIFIED DCFROR

A way to modify the cash flow stream for a project with a T-type curve to obtain a unique value of $\tilde{R}$ has been proposed [1]. As we now show, the value obtained is not satisfactory for decision-making. With the modification the negative cash flows at the end of the project are discounted back to time zero using i_m. Denote the value obtained by I_{oo}. $\tilde{R}$ is then calculated using as initial investment the actual investment, I_o plus I_{oo}. The value obtained is the *modified DCFROR*, R_m. For Project T depicted in Figure 4 with $i_m = 0.15$ we have

$$(23)\quad I_{oo} = \frac{80}{0.15}\,[1 - \exp\{-0.15*5\}] * \exp\{-0.15*7\} = 98.5$$

(We refer to the first year with negative cash flow, here 7, as the initial terminal year.) The NPV equation becomes

$$(24)\quad NPV(i) = -98.5 - \frac{45}{i}*\left[1-\exp\{-2i\}\right] + \frac{80}{i}*\left[1-\exp\{-5i\}\right]*\exp\{-2i\}$$

Setting $NPV(i) = 0$ and solving gives R_m = 19.5%. Since $R_m > i_m$, the inference is that Project T is acceptable, consistent with the conclusion reached earlier using $\tilde{G}$, *PVR* and *NPV*.

What interpretation can we place on modified DCFROR? Is it the rate of return on unamortized investment? If so, what investment? To answer these questions we turn, as we did for $\tilde{R}$, to the cumulative cash flow, *CCF(t)*. With the modified investment, I_{oo}, the counterpart of Eq.(11) for Project T is

$$(25)\quad dCCF = -I\,dt + i*CCF(t)\,dt\,;\quad CCF(0) = -I_{oo} \quad \left(0 \lhd t \lhd 2\right)$$

Thus for the 1st two years cash tied up in the project increases, i.e., *CCF* becomes more negative, at the rate of investment, *I* \$/yr, plus the interest paid on the current value of *CCF(t)*, %/yr*\$. Integrating Eq(25) gives for *CCF(2)*:

$$(26)\quad CCF(2) = -I_{oo}*\exp\{2i\} - \frac{I}{i}*\left[\exp\{2i\}-1\right]$$

After year 2 with cash flow, A \$/yr, the equation for *CCF(t)* becomes:

$$(27)\quad dCCF/dt = A + i*CCF(t)\;;\quad \left(2 \lhd t \lhd 7\right)$$

Using CCF(2) from Eq.(26) as initial condition, the solution to Eq.(27) is

$$(28)\quad CCF(t) = -I_{oo}*\exp\{it\} - \frac{I}{i}*\left[\exp\{2i\}-1\right]*\exp\{t-2\} + \frac{A}{i}*\left[\exp\{t-2\}-1\right];\ \left(2 \lhd t \lhd 7\right)$$

If in Eq.(28) we set $t = 7$, $I_{oo} = 98.5$, $I = 45$, $A = 80$ and multiply both sides by $exp(-7i)$, we obtain

$$(29)\quad CCF(7)*\exp\{-7\} = -98.5 - \frac{45}{i}*[1-\exp\{-2i\}] + \frac{80}{i}*[1-\exp\{-5i\}]*\exp\{-2i\}]$$

The rhs of Eq. (29) is identical to Eq.(24) solved earlier to obtain R_m. Taken together Eqs.(23)-(29) show that R_m is the rate of return on unamortized investment, where the investment being amortized is I_{oo} plus the actual investment in the project. With i equal R_m this investment is fully amortized at $t = 7$. I_{oo} is invested at $t = 0$ at interest rate i_m, and the future value of this investment is just sufficient to pay the negative cash flows at the end of the project.

Modified DCFROR In Practice

Although R_m resolves the problem of dual rates of return, to obtain smoothly consistent results with the technique requires further consideration, as the following example illustrates. Consider the cash flow streams which result when Project T's negative annual cash flow, A, during years 7-12 (see Figure 4) is allowed to vary as shown in Table IV.

Table IV
Sensitivity of Economic Yardsticks
Modified DCFROR, R_m, with Terminal Year n = 7, i_m = 15%

Case	\$A,7-12	NPV(0)	NPV(15)	$\tilde{G}$	PVR	$\tilde{R}$	I_{oo}	R_m
1	10	360	143	20.2%	1.84	48.5%	−12.3	59.9%
2	0	310	131	19.8%	1.68	48.1%	0.0	48.1%
3	−10	260	118	19.4%	1.52	47.7%	12.3	41.5%
4	−36	130	86	18.3%	1.10	46.5%	44.3	30.5%
5	−50	60	69	17.7%	0.89	45.7%	61.5	26.3%
6	−62	0	54	17.0%	0.70	0.0,45.0%	76.3	23.3%
7	−80	−90	32	16.1%	0.42	7.1,43.8%	98.5	19.5%

Consider the values in columns 2 and 3. As per the definitions given in Figure 5, cases 1-3 are *N-type* (because *NPV(i)* decreases monotonically as i increases). Case 4 is on the border between *N-type* and *U-type*. (The plot of *NPV(i)* vs i has zero slope at $i = 0$.) Case 5 is *U-type*, case 6 is on the dividing line between *U-type* and *T-type*, and case 7 corresponds to Curve *T*.

Although they decrease smoothly as the cash flow becomes progressively less favorable, the values of *NPV* in the fourth column indicate the investment to be acceptable for all cases. Both *NPV* and *PVR* indicate a large variation in the desirability of these investment cases. $\tilde{G}$ decreases smoothly as the cash flow becomes less attractive, but the indicated quality reduction is to a much smaller degree; from highest to lowest $\tilde{G}$ drops by only about one-third, whereas both *PVR* and *NPV* fall by nearly a factor of five. The investor needs to assimilate this difference in sensitivity, which is typical, in order to properly temper judgment based on these yardsticks.

For the five cases yielding a unique $\tilde{R}$, this yardstick varies only slightly suggesting almost no difference in preference. Interestingly enough, for cases 6 and 7 the larger of the two $\tilde{R}$ appears to be a smooth extension of the values for cases 1-5. Comparison of R_m for cases 6 and 7 to the $\tilde{R}$ for cases 1 and 5 reveals a troubling inconsistency. With A = − 50 $\tilde{R}$ = 45.7%, whereas with A = − 62, R_m = 23.3%. It is certainly not reasonable to construe that the latter project is so much poorer than the former. Hence, we conclude that comparing $\tilde{R}$ to R_m does not provide a valid measure of the relative desirability of two investment alternatives.

If R_m is computed using a uniform procedure for all cases in Table IV, a smooth and consistent measure of investment quality is obtained even when, as in case 1, the cash flows during the terminal years are positive, so that I_{oo} is negative. $I_{oo} < 0$ can be interpreted as money (1) borrowed at $t = 0$ at interest rate i_m, (2) used to reduce the initial investment, and (3) amortized by the cash flows from the project in years 7-12. As case 2 illustrates, $\tilde{R}$ and R_m are identical when cash flow during the terminal years is zero.

THE BOTTOM LINE

In summary, we have shown the following about these four investment measures:

1. *PVR depends only on i_m.*
2. *$\tilde{G}$ depends upon i_m and the terminal year, n, used to calculate future worth, F_n.*
3. *R_m depends upon i_m and initial terminal year used to calculate investment adjustment, I_{oo}.*
4. *$\tilde{R}$ does not depend upon any arbitrarily specified parameters.*
5. *PVR provides a smoothly consistent measure for all investment projects.*
6. *When a common terminal year is used, $\tilde{G}$ also provides a smoothly consistent measure.*
7. *$\tilde{G}$ can be determined analytically from PVR, and vice versa.*
8. *$\tilde{R}$ can be used only when the cash flow profile is N-type.*
9. *R_m eliminates dual rates of return and provides a smoothly consistent investment measure if a suitable common initial terminal year is used for all projects. However, the required adjustment process is arbitrary and inconvenient, effectively eliminating the utility of R_m for decision making.*
10. *Values of $\tilde{R}$ and R_m for a project are not, in general, comparable. The hybrid process; use R_m only for projects which yield dual rates of return and $\tilde{R}$*

for all other projects, does not provide a satisfactory, consistent ranking of the desirability of investment alternatives.

WHAT INVESTMENT SHOULD BE USED?

At first blush the answer to this question seems simply and obviously "*The investment is the money spent to undertake the project*". This transparency notwithstanding two situations in which care must be taken in specifying investment in order to avoid generating fallacious evaluation signals are when

1. *sunk costs are involved,*
2. *a year or two with positive net cash flow is interspersed within the (early) period during which investment takes place.*

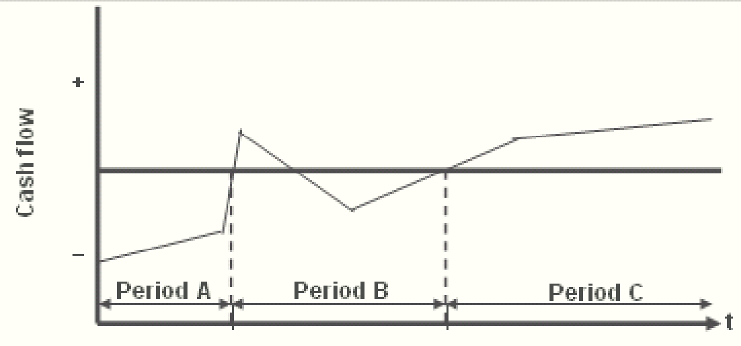

Figure 6 Cash Flow To Include In Investment

Here we consider only the second situation. Clearly, investment includes the sum of all negative cash flows up until the time when cash flow first becomes positive. But what about configurations such as that shown in Figure 6 in which cash flow after changing from negative to positive, becomes negative, and positive again?

With cash flow thus configured smooth and consistent economic measures are obtained if investment is defined according to the following rule:

- ***Include in PWI the discounted sum of all cash flows during Period B in Figure 6 only if this quantity is negative.***

Why this rule? During Period B the early positive cash flows, plus interest accrued at i_m until these funds are expended, are used to pay the negative cash flows later in the period. If the discounted sum is negative, the period B credits are insufficient to offset the negative cash flows, and a drain on the cash till occurs. The cash required is the discounted sum of the cash flows during Period B.

The economic evaluations for the projects tabulated in Table V illustrate the proper way to deal with this situation. The calculations were made using $i_d = 15\%$, but the conclusions are independent of what rate is used. For all of these projects period A spans years 1-2 and period B spans years 3-5.

- *PWI* for Projects A, B, & C differ because the discounted sum of the cash flows during period B is negative.

- For Project D this sum is zero. If the discounted sum of the cash flows in period B is zero, the future worth of these cash flows is also zero; for this reason F_{10} for Project D is the same as that for Projects A-C.
- For Projects E, F, G & H this sum is positive. Hence, for each of the latter five projects *PWI* is the discounted sum of (negative) cash flows in years 1-2.

Table V
Example Projects Illustrating How To Calculate Initial Investment Correctly

	Project							
Year	A	B	C	D	E	F	G	H
A-----1	− 240	− 240	− 240	− 240	− 240	− 240	− 240	− 240
A-----2	− 100	− 100	− 100	− 100	− 100	− 100	− 100	− 100
B-----3	100	200	200	200	200	200	100	100
B-----4	0	− 100	− 100	− 100	− 100	− 100	0	0
B-----5	− 200	− 200	− 160	− 154	− 150	− 100	− 100	0
C-----6	100	100	100	100	100	100	100	100
C-----7	300	300	300	300	300	300	300	300
C-----8	300	300	300	300	300	300	300	300
C-----9	200	200	200	200	200	200	200	200
C-----10	100	100	100	100	100	100	100	100
PWI(.15)	336	326	306	303	303	303	303	303
F_{10}	1390	1390	1390	1390	1398	1504	1464	1676
$\tilde{G}$,%	14.20	14.50	15.13	15.23	15.29	16.02	15.75	17.10
PVR(.15)	− .077	− .049	.014	.024	.030	.106	.078	.234
NPV(.15)	− 26	− 16	4	7	9	33	24	71

Considering Projects A-F in alphabetic order, it is intuitively obvious that each project is preferable to the project immediately preceding it in the tabulation. For these projects, the values of $\tilde{G}$ reflect this preference in a smoothly consistent manner, even as we pass by the transitional Project D where cash flows in period B are moving from the investment to the future worth side of the defining equation for $\tilde{G}$. This consistency is unaffected by the fact that Projects A & B do not meet the investment standard whereas Projects C, D, E & F do. Finally, taken as a group Projects A, G & H provide an example of a smooth transition in $\tilde{G}$ as the negative cash flow in period B disappears.

The tabulation also shows that by following the rule specified for determining *PWI*, consistent values of *PVR* are obtained. This result is, of course, no surprise, since *PVR* and $\tilde{G}$ are analytically related by Eq.(22).

HOW TO TREAT INFLATION

What modifications should be made in economic evaluations to allow for inflation? In an inflationary world the quantity of goods which a given (nominal) amount of money will buy decreases with time. Or, stated another way, the dollar prices of all goods increase with time The inflation rate, *f*, measures how fast money decreases in buying power. Although *f* is ordinarily given in percent/year, in calculations *f* is a fraction. Quantitatively, the real buying power of \$1 decreases by an amount $exp(-f*t)$ in t years. Thus, if $f = 0.10$, two years from now *\$100* will buy only as many goods as $100*exp(-0.1*2) = \$81.87$ will buy today. The deflated value of *\$100* received two years from now is *\$81.87*.

Because of inflation's effect on buying power over time, we must differentiate between the quantity of money changing hands at a point in time to the buying power of that money at some earlier specified date. We call the cash transaction current or nominal dollars and its earlier buying power real or constant dollars. For the above example *\$81.87* is the real or constant dollar buying power, relative to a base point two years earlier, of *\$100* received currently.

The modifications to economic evaluation procedures to treat inflation are more in concept than in methodology. The fundamental thesis is to measure profitability of a venture in constant dollars. To use current dollars instead is to delude ourselves, as the following example shows. Suppose we make an investment of *\$10,000* at $t = 0$ which yields us *\$1000/year* for *5* years, at which time the salvage value of our investment is *\$10,000*. Clearly, this investment yields $\tilde{R} = 10\%$ measured in current dollars. Suppose that we invest the cash flow from the project at *8%*, also in nominal terms. Then at the end of *5* years our actual dollar holdings will be

$$F_5 = 10,000 + 1000*[exp(0.08x5) - 1]/0.08 = \$16,148 \text{ Current Dollars}$$

Suppose that during this 5 year period the inflation rate is $f = 9\%/year$. With this inflation rate the buying power of our future holdings measured in real dollars at $t = 0$ is

$$F_5 = 16,148*exp(-0.09x5) = \$10,296 \text{ Real Dollars}$$

so that our buying power increases by an almost negligible amount.

Note that this same result is obtained from

$$(30)\quad 10000*\exp\{-0.09*5\} + \int_0^5 [1000*\exp\{-0.09*t\}]*\exp\{-0.01*(5-t)\}\,dt = 10296$$

This equation states precisely the way in which inflation affects the real rate of return. The first term is the real dollar buying power of the salvage value, and the integral is the real dollar buying power at the end of year *5* of the reinvested interest received from the project. The term *1000*exp(−0.09t)*dt* is the real dollar buying power of the interest received between *t* and *t + dt*. The term *exp(−0.01(5−t))* represents the loss in buying power of this increment of interest between the time it is received and the end of the project at *t = 5*. This increment loses buying power at the rate of *1%/year* because it is invested at *8%/yr* whereas the inflation rate is *9%/year*.

If we calculate $\tilde{G}$ each of these situations, we have

Current: ***10,000*exp(*** $\tilde{G}c$ ****5) = 16,148 ▶*** $\tilde{G}c$ ***= 9.58%.***

Real: ***10,000*exp(*** $\tilde{G}r$ ****5) = 10,296 ▶*** $\tilde{G}r$ ***= 0.58%.***

Subtracting we have ***9.58% − 0.58% = 9.0%*** which illustrates that

$$(31)\quad \tilde{G}_c - \tilde{G}_r = f$$

Eq.(31) contains the essence of inflation's effect on economic evaluations: The current dollar rate of return is equal to the real rate of return plus the inflation rate. This same result holds for $\tilde{R}$, as is easily demonstrated for this example. The economic yardstick that is meaningful to us is the real rate of return. The above example shows that we can obtain this desideratum in either of two ways:

1. *Calculate the rate of return using current dollars throughout and subtract the inflation rate from the result.*
2. *Calculate the real rate of return directly using constant dollars throughout.*

From the foregoing, we conclude that economic evaluations in an inflationary world are performed via the following steps:

1. *Estimate costs and revenues for the life of the project in today's prices.*
2. *Escalate costs and revenues to obtain the time profile of these transactions in current dollars. By the term escalate we imply that costs and revenues may inflate at different rates, none of which is necessarily equal to the rate of inflation.*
3. *Calculate current dollar yardsticks directly from these cash flows.*
4. *For constant dollar evaluation deflate the escalated values at the estimated inflation rate, f, to obtain constant dollar values.*

Eq.(31) concisely expresses the effect of inflation on calculated values of $\tilde{R}$ and $\tilde{G}$. An investor's desired minimum rate of return, i_m, is affected in exactly the same way as $\tilde{R}$. Thus, to establish a meaningful investment standard an investor must clearly state whether his value of i_m is measured in real or current dollars. Specifying i_m in real terms is the sounder option, since doing so yields a measure of rate of increase in wealth that is independent of inflation rate. The form of the decision criterion is exactly alike for the two specifications, e.g., with $\tilde{R}$ the requirement is

- ***An investment is satisfactory if $\tilde{R}_c > i_{mc}$ or if $\tilde{R}_r > i_{mr}$ where $i_{mc} - i_{mr} = f$. Replacing $\tilde{R}$ with $\tilde{G}$ yields the criterion for this measure of earnings rate.***

To obtain valid values of *NPV* for investment decision-making one uses i_{mc} with escalated cash flows and i_{mr} with deflated cash flows. The values of *NPV* calculated in these two alternate ways are identical, so that the requirement that $NPV > 0$ for an acceptable investment yields the same decision for each calculation procedure. The calculation of *PWI* must be adjusted in precisely the same way as *NPV*; i_{mc} is used with escalated net investments and i_{mr} is used with the deflated values. In either case, the same value of *PWI* is obtained. Thus, since $PVR = NPV / PWI$, the same value of this investment yardstick is obtained using either current or constant dollars provided that inflation is treated properly.

HOW TO TREAT DEPRECIATION WITH INFLATION

With inflation, a second important adjustment in evaluation procedures is proper treatment of depreciation. Depreciation is the analytical procedure used to charge capital costs against the revenues generated through use of the associated capital goods. Because, after purchase in some year, capital facilities are used to generate output over several years, a considerable degree of arbitrariness exists as to how much the charge for the capital goods consumed in any particular year should be.

It is inarguable that at the end of a year in which a capital good is worn out and abandoned, the sum of all depreciation charges for this good in this and previous years should equal the original purchase cost of the item. Upon closer inspection, however, it becomes apparent that to truly reflect the cost of the capital good, the sum of the depreciation charges must exceed the item's original purchase cost. When the item wears out, if the business is to continue, the good must be replaced. In an inflationary environment the new purchase cost will exceed the old. Thus, if the sum of the depreciation charges simply equals the original purchase cost, the cost of the capital goods used in production will always be understated. (This inequity can be offset by allowing the use of accelerated depreciation procedures that cause the depreciation charge to be higher than proportionate early in the facility's life, and less than proportionate thereafter.) Since a portion of the costs are understated, it follows that net income is overstated. Although such overstatement may seem attractive when present business performance is being measured, this apparent benefit is illusory. Overstating net income causes income taxes to go up. As a result, a portion of the revenues, which should have been used to pay capital costs, are paid as taxes instead. This siphoning effect has a negative impact on long- run business profitability and thus acts to depress industrial vitality.

We consider two examples to illustrate how depreciation charges based on historical costs should be treated in an economic evaluation and to show the degree to which earnings are overstated. First, consider a capital investment, *Io = $100,000* (today's dollars) made during year *1* to establish production yielding (constant dollars) gross revenues of *$50,000/year* in years *2 − 11*. Out-of- pocket expenses are *$20,000/year* (also in constant dollars). In this case either unit-of-production or straight line (life = *10* years) depreciation results in charging each unit of revenue with an equal amount of the original capital cost. Assume income tax rate, $t_a = 40\%$, inflation rate = *10%/year*, and revenues and expenses escalate at the inflation rate.

The depreciation charge is *$10,000/year* in years *2 − 11*, inclusive. Since this is the amount actually charged in each year, it is in current dollars. Subtracting expenses from gross revenues yields operating margin before depreciation of $30,000/year; this amount is in constant dollars. Since income taxes are based upon current dollars, income must be determined on this basis. Pre-depreciation current dollar income is *30,000*exp(0.10*t)*. Subtracting depreciation and multiplying the result by *0.6* (*1* minus the tax rate) gives after-tax net income in thousands of current dollars of *0.6*[30*exp(0.10*t) - 10]*. Adding depreciation to after-tax net income gives net cash flow as

$$(32)\ ncf(t) = 0.6*\lfloor 30*\exp\{0.10*t\} - 10\rfloor + 10\ ; \quad (1 \le t \le 11)$$
$$= 18*\exp\{0.10*t\} + 4$$

(This expression illustrates the rule that each dollar of depreciation increases cash flow by t_a current dollars, where t_a = tax fraction, i.e., a depreciation of *10* increases *ncf(t)* by *4*.)

To determine $\tilde{R}c$ we set

$$(33)\ NPV = -PWI + \int_0^{t_n} ncf(t)*\exp\{-\tilde{R}_c*t\}dt = 0$$

which using Eq(32) gives

$$(34)\ PWI = \int_1^{11} [18*\exp\{0.1*t\} + 4]*\exp\{-\tilde{R}_c*t\}dt$$

Integrating the rhs of Eq(34):

$$(35)\ PWI = \frac{18}{\tilde{R}_c - 0.1} * \left[1 - \exp\{-(\tilde{R}_c - 0.1)*10\}\right] * \exp\{-(\tilde{R}_c - 0.1)\}$$
$$+ \frac{4}{\tilde{R}_c} * \left[1 - \exp\{-\tilde{R}_c * 10\}\right] * \exp\{-\tilde{R}_c\}$$

To be consistent we convert initial investment, *Io*, which is spread uniformly over year 1, to its current dollar equivalent by multiplying by *exp(0.10*t)*, The resulting expression for NPV is

$$(36)\ PWI = \int_0^1 100 * \exp\{-(\tilde{R}_c - 0.1)*t\}dt = \frac{100}{\tilde{R}_c - 0.1} * \left[1 - \exp\{-(\tilde{R}_c - 0.1)\}\right]$$

Substituting Eq.(36) into Eq.(35) and solving the resulting equation by trial-and-error gives $\tilde{R}c$ *= 24.7%/year.*

Deflating the current dollar cash flow, i.e., *ncf(t)*exp(−0.10*t),* the equation giving real rate of return is

$$(37)\ PWI = \int_1^{11} \left[18 + 4*\exp\{-0.1*t\}\right] * \exp\{-\tilde{R}_r * t\}dt$$

The real dollar equivalent to Eq(36) for PWI is

$$(38)\ PWI = \int_0^1 100 * \exp\{-\tilde{R}_r * t\}dt = \frac{100}{\tilde{R}_r} * \left[1 - \exp\{-\tilde{R}_r\}\right]$$

Since by Eq(31)

$$(39)\ \tilde{R}_c - \tilde{R}_r = 0.1$$

Eq(36) and (38) both give PWI = 93 with $\tilde{R}c$ = 0.247 and $\tilde{R}r$ = 0.147. Substituting Eq.(38) into Eq.(37) and solving by trial-and-error gives $\tilde{R}r$ *=14.7%/year*, a value which satisfies Eq.(39).

Since depreciation deductions are stated in current dollars, these amounts are subtracted directly in computing net cash flow in current dollars, as was done in Eq.(32). However, if real dollars are used in the calculations, depreciation deductions must be deflated, as was done in Eq.(37).

We now examine this example further to see how much capital costs are understated and earnings overstated when depreciation charges are based on historical cost. Assume that the replacement cost of the capital facility increases at the rate of inflation. Given this assumption, if depreciation deductions were to increase at the rate of inflation, the cumulative deduction over an extended period of time would approximately average out to the sum of actual capital outlays. At any time depreciation equals *10*exp(0.10*t)* so that instead of Eq.(32) net cash flow in current dollars is given by

$$(40)\ ncf(t) = 22*\exp\{0.10*t\}\ ; \quad (1 \le t \le 11)$$

and the determining equation for $\tilde{R}c$ becomes

$$(41)\ PWI - \frac{22}{\tilde{R}_C - 0.1} * \left[1 - \exp\{-(\tilde{R}_C - 0.1)*10\}\right] * \exp\{-(\tilde{R}_C - 0.1)\} = 0$$

with *PWI* given by Eq.(36) as before. Solution of Eq.(41) gives $\tilde{R}c$ = *26.3%/year*. Clearly, $\tilde{R}r$ *= 16.3%/year* in this case.

Thus, we see that for this example increasing depreciation deductions at the rate of inflation increases the real rate of return by *1.6%*, from *14.7%* to *16.3%.* Such an increase is to be expected since the increased deduction increases net cash flow by reducing income tax. The reduction in income tax, which equals t_a multiplied by the reduction in net income, is given by

0.4*[30*exp(0.1t) - 10 - 22exp(0.1t)] = 4*[exp(0.1t) - 1]

In current dollars total income tax paid during the life of the project is lowered by

$$(42)\ \int_1^{11} 4*\left[\exp\{0.1\} - 1\right] dt = \frac{4}{0.1}*\left[\exp\{0.1*11\} - \exp\{0.1\}\right] - 40 = 36.0$$

which is a reduction of approximately 19%. In a similar manner it can be shown that with depreciation based on historical capital cost, cumulative after-tax net income from the project is 281.8, whereas with depreciation based on replacement cost the corresponding value is 227.9. Hence, in the former case net earnings are overstated by 24%. (If 281.8 is used as the divisor in computing the percentage overstatement, the value obtained is the same as for the income tax reduction, i.e. 19%.) This overstatement results in total income tax payments being 24% greater than sound economic reasoning indicates they should be.

As a final consideration let us determine the amount by which NPV is reduced as a result of basing depreciation on historical costs. To determine this quantity we must first specify a value for i_m. Assume i_{mr} *= 10%/year*, for which i_{mc} *= 20%/year*. Let us determine *NPV* using current dollars, so that i_{mc} is the discount rate used. With historical cost depreciation *NPV* is the nonzero value obtained from Eq.(33) with $i_{mc} = 0.2$; *ncf(t)* is given by Eq.(32) and *PWI* by Eq.(34). Substituting numerical values yields *NPV = 22.0.* With depreciation based upon replacement cost *ncf(t)* is given by Eq.(40). Substituting this expression into Eq.(41), again using $i_{mc} = 0.2$, and once more obtaining *PWI* from Eq.(33), the result is *NPV = 30.7*. Thus, for this discount rate using historical cost depreciation causes *NPV* to be over 25% less than would be the case with replacement cost accounting.

SUMMING UP

Making economic evaluations is an important, difficult, and never-ending task in the oil and gas business. Before any significant activity is undertaken, its likely cost must be carefully weighed against the probable benefits with due consideration of the uncertainty involved. All techniques aside, proper treatment of uncertainty requires that all data and assumptions be carefully scrutinized. The downward drift of oil and gas prices and the inevitable upward drift of costs amplify the need for such scrutiny today. Simply stated, project experts need to focus their attention on judging the underlying performance of the project. Hopefully, by clarifying some issues about evaluation methodology, this paper will prevent time from being spent on distracting concerns about the evaluation techniques employed.

Relative to inflation two points considered in the text are worthy of further amplification. The first centers around the difference between the real and the current rate of return. We showed that these differ by the inflation rate, a result familiar to most evaluation analysts. In spite of this familiarity, we have a vague suspicion that most companies do not properly recognize this difference when they establish their investment standards. In the oil patch discount rates used on investment projects probably, in the main, vary from *15 to 25* percent, with even higher values used on occasion for projects considered to have an unusually large degree of risk. Given the magnitude of the discount rate, this i_m must be specified in current dollars. Subtracting an inflation rate of *10%* indicates a targeted real rate of return plus a risk premium of *5 to 15%*. Since financial theorists tell us that the risk-free real cost of capital is maybe *3%*, we arrive at a risk premium of *2 to 13%*. This schema seems sensible until one recalls that the same discount rates were also used when inflation rate was *5%* and when it was *15%*. And during this period average rate of return on equity in the oil business varied between *12%* and *14%*. It seems to us that a corporation's objectives would be more clearly specified by stating a targeted real rate of return, defining an operational way of determining the risk premium to be assigned to various types of projects, and letting its economists specify the inflation rate to be added on.

The second point worthy of examination is the question of escalation. Certainly, we are all aware that a few years back numerous investment decisions in the oil patch, which have turned out to be lemons, were triggered by what have now proven to be overly optimistic escalations of oil and gas prices. Nowadays, it is fashionable to enter an oil price profile with zero, or perhaps even negative, escalation for a few years followed by an escalation rate equal to or slightly greater than the inflation rate during the rest of the project's years. As far as costs are concerned, these last few years of retrenchment in the oil patch have brought some significant declines in costs, but common sense tells us that costs must escalate at the inflation rate if the suppliers are to stay in business. This latter requirement is the point we would like to stress.

In making projections beyond two or three years into the future it becomes very tenuous, indeed, to hypothesize that escalation rates of costs and revenues will deviate by much from the inflation rate for very long. The inflation rate is simply an average of the escalation rates of prices of all things sold in the economy. For the escalation rate of any major category of items to deviate from the inflation rate by as much as *0.5%* for any extended period of time, it is necessary that significant shifts in the sub-economy underlying those items occur. Sometimes, technological advances cause such shifts, as we are witnessing in the downward drift of manufacturing costs for computer chips today. Fundamental supply shortages may cause prices of some items to move to new elevated equilibrium levels; such a shift is what we divined as effecting oil and gas prices during the past decade. The shortsightedness of these divinations is now clearly before us. Not only is there sufficient capacity in the world to more than supply all the oil and gas that people want to buy at present prices, but there are still lots of places around the world where additional oil and gas can be found and put on production at a cost which present prices will amortize with a very handsome return. Our caveat is to be very careful about overplaying the escalation game, one way or the other.Perhaps the procedure that was used years ago before inflation became a major factor in our thinking is the most prudent one: Perform evaluations in real dollars (Use present prices and costs assuming that all will inflate at a common rate.) after deflating depreciation deductions at the projected inflation rate.

NOMENCLATURE

A = rate of cash flow from an investment, $/year.

CCF(t) = cumulative cash flow from an investment at time t, $

CCF(0) = cumulative cash flow from an investment at t = 0 (equals initial investment, I_o), $

CCF(n) = cumulative cash flow from an investment at t = n, $.

d = differential operator.

DCFROR = $\tilde{R}$ = discounted cash flow rate of return for an investment, %/yr.

$DCFROR_c$ = $\tilde{R}c$ = DCFROR measured in current dollars, %/yr.

$DCFROR_r$ = $\tilde{R}r$ = DCFROR measured in real dollars, %/yr.

F = future worth of some amount of money, $.

GROR = $\tilde{G}$ = growth rate of return for an investment, %/yr.

$GROR_c$ = $\tilde{G}c$ = GROR measured in current dollars, %/yr.

$GROR_r$ = $\tilde{G}r$ = GROR measured in real dollars, %/yr.

i, i_d = discount or interest rate, %/yr.

i_m = investor's minimum targeted earnings rate, %/yr.
i_{mc} = i_m measured in current dollars, %/yr.
i_{mr} = i_m measured in real dollars, %/yr.
I = rate of investment, $/yr.
I_0 = initial investment at t = 0, $.
I_{oo} = investment adjustment used in modified DCFROR procedure.
n = life of an investment project, years.
NPV = discounted present value (net present worth) of an investment project, $.
ncf(t) = rate of net cash flow from an investment project at time t, $/yr.
P_o = present worth at t = 0 of some amount of money, $.
PWI = present worth of the investment in at project, $.
PVR = ratio of NPV to PWI for a project.
t = time, years.

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